Problem 10-04 Einstein equation, effective particle dimension

For the diffusion coefficient of particles having the shape of an elongated paraboloid in the liquid medium (density ρ_o = 0.934 g cm^{-3} and viscosity η_o = 0.8346 mPa s) was found the value $D = 8.6\cdot10^{-7}$ cm^2 s^{-1} at the temperature of 21 °C. Calculate

(a) friction coefficient,

(b) effective radius of a particle, i.e. the radius of a spherical particle which diffuse in the same manner as studied non-spherical particle

(c) estimate the mass of a single particle and the mass of one mole of the these particles. The density of particles is ρ = 1.13 g cm^{-3}.

[(a) $f = 4.72\cdot10^{-11}$ kg s^{-1} (b) $r_{ef} = 3\cdot10^{-9}$ m, (c) $m = 1.278\cdot10^{-22}$ kg, M = 76.96 kg mol^{-1}]

Řešení:

T = 294.15 K
$D = 8.6\cdot10^{-7}$ cm^2 s^{-1} = $8.6\cdot10^{-11}$ m^2 s^{-1}
ρ_o = 0.934 g cm^{-3}
η_o = 0.8346 mPa s
ρ = 1.13 g cm^{-3} = 1139 kg m^{-3}

(a) Friction coefficient – from Einstein equation: $D = \dfrac{k_B\,T}{f}$

$$f = \frac{k_B\,T}{D} = \frac{1.38\cdot10^{-23}\cdot294.15}{8.6\cdot10^{-11}} = 4.72\cdot10^{-11}\ \text{kg s}^{-1}$$

$$\left[\frac{(\text{J K}^{-1})\cdot\text{K}}{\text{m}^2\,\text{s}^{-1}} = \frac{(\text{kg m}^2\,\text{s}^{-2}\,\text{K}^{-1})\cdot\text{K}}{\text{m}^2\,\text{s}^{-1}} = \text{kg s}^{-1}\right]$$

b) Spherical particles having radius r_{ef} and the same diffusion coefficient as the non-spherical ones have also the same friction coefficient. According to the Stokes equation

$$f = 6\,\pi\,\eta_o\,r_{ef}$$

$$r_{ef} = \frac{f}{6\,\pi\,\eta_o} = \frac{4.72\cdot10^{-11}}{6\,\pi\cdot8.346\cdot10^{-4}} = 3\cdot10^{-9}\ \text{m}$$

$$\left[\frac{\text{kg s}^{-1}}{\text{Pa s}} = \frac{\text{kg s}^{-1}}{\underbrace{\text{kg m s}^{-2}}_{\text{N}}\,\text{m}^{-2}\,\text{s}} = \text{m}\right]$$

c) $m = V\cdot\rho = \frac{4}{3}\pi\,r^3\rho = \frac{4}{3}\pi\cdot(3\cdot10^{-9})^3\cdot1130 = 1.278\cdot10^{-22}$ kg

$M = m\cdot\boldsymbol{N_A} = 1.278\cdot10^{-22}\cdot6.022\cdot10^{23} = 76.96$ kg mol^{-1}