Problem 10-02 Einstein equation, particle dimensions from diffusion coefficient

Spherical micelles of a surfactant are large particles (in comparison with the molecules of the dispersion medium) of density 1022.1 kg m^{-3}. Their diffusion coefficient at the temperature of 23.2 °C and pressure of 0.1 MPa in the dispersion medium of density 0.922 g cm^{-3} and viscosity 1.13 mPa s was found to be $4.8 \cdot 10^{-11}$ m^2 s^{-1}. Calculate

(a) the diameter of a micelle, (b) mass of one mole of micelles.

[(a) d = 8 nm; (b) M = 165 kg mol^{-1}]

Solution:

T = 296.35 K
p = 0.1 MPa

η_o = 1.13 mPa s = $1.13 \cdot 10^{-3}$ Pa s
ρ_o = 0.922 g cm^{-3} = 922 kg m^{-3}
D = $4.8 \cdot 10^{-11}$ m^2 s^{-1}
ρ = 1022.1 kg m^{-3}

Einstein equation for diffusion coefficient: $D = \frac{k_B T}{f}$

Stokes equation for friction coefficient: $f = 6 \pi \eta_o r$

$$D = \frac{k_B T}{6 \pi \eta_o r}$$

(Stokes-Einstein equation)

(a) $d = 2r = 2\frac{k_B T}{6 \pi \eta_o D} = \frac{1.38 \cdot 10^{-23} \cdot 296.35}{3 \pi \cdot 1.13 \cdot 10^{-3} \cdot 4.8 \cdot 10^{-11}} = 8 \cdot 10^{-9} = 8$ nm

(b) $M = N_A \cdot m = N_A \cdot \frac{4}{3} \pi r^3 \cdot \rho = 6.022 \cdot 10^{23} \cdot \frac{4}{3} \pi \cdot \left(\frac{8 \cdot 10^{-9}}{2}\right)^3 \cdot 1022.1 = 165$ kg mol^{-1}