Problem 10-03 Einstein equation for diffusion coefficient

Calculate the diffusion coefficient of spherical particles of a protein (density ρ = 1.228 g cm^{-3}, molar mass M = 68 kg mol^{-1}) at the temperature of 302 K and the pressure of 99.3 kPa in liquid medium of density ρ_o = 0.834 g cm^{-3} and viscosity η_o = 0.932 mPa·s.

[$D = 8.5 \cdot 10^{-11}$ m^2 s^{-1} ($r = 2.8 \cdot 10^{-9}$ m)]

Solution:

M = 68 kg mol^{-1}
T = 302 K
η_o = 0.932 mPa·s = $9.32 \cdot 10^{-4}$ mPa·s
ρ_o = 0.834 g cm^{-3} = 834 kg m^{-3}
ρ = 1.228 g cm^{-3} = 1228 kg m^{-3}

$$V = \frac{4}{3} \cdot \pi \cdot r^3 = \frac{M}{\rho \cdot N_A} \quad \Rightarrow \quad r = \sqrt[3]{\frac{3 \cdot M}{4 \cdot \pi \cdot \rho \cdot N_A}} = \sqrt[3]{\frac{3 \cdot 68}{4 \cdot \pi \cdot 1228 \cdot 6.022 \cdot 10^{23}}} = 2.8 \cdot 10^{-9} \text{ m}$$

$$\left[\left(\frac{\text{kg mol}^{-1}}{(\text{kg m}^{-3}) \cdot (\text{mol}^{-1})} \right)^{1/3} = \text{m} \right]$$

$$D = \frac{k_B \cdot T}{f} = \frac{k_B \cdot T}{6 \pi \cdot \eta_o \cdot r} = \frac{1.38 \cdot 10^{-23} \cdot 302}{6 \cdot \pi \cdot 9.32 \cdot 10^{-4} \cdot 2.8 \cdot 10^{-9}} = 8.5 \cdot 10^{-11} \text{ m}^2 \text{ s}^{-1}$$

$$(f = 6 \cdot \pi \cdot \eta_o \cdot r) \qquad \left[\frac{(\text{J K}^{-1}) \cdot \text{K}}{(\text{N m}^{-2} \text{ s}) \cdot (\text{m})} = \text{m}^2 \text{ s}^{-1} \right]$$

$\text{N} = \text{J m}^{-1}$