Problem 12-03 Membrane equilibria

Left compartment of the dialysis cell was filled with an aqueous solution of the sodium salt of protein (molar mass 63 kg mol^{-1}), the concentration of which was 75.6 g dm^{-3}. At pH = 7.4 the protein molecule carries six charges, $(Na^+)_6P^{6-}$. Into this solution was then added sodium chloride in such amount that its concentration was 0.18 mol dm^{-3}. The right compartment was filled with pure water. Both compartments are of the same volume and they are separated by a semipermeable membrane which lets pass the low-molecular species, but not the protein. Calculate the equilibrium concentrations of all ions in both compartments.

[$(c_{Na^+})_{Left}$ = 95.435 mmol dm^{-3}; $(c_{Cl^-})_{Left}$ = 88.235 mmol dm^{-3}; $(c_{Na^+})_{Right} = (c_{Cl^-})_{Right}$ = 91.765 mmol dm^{-3}]

Solution:

NaCl ≡ N, Na_6P ≡ P (protein)

$(Na^+)_6P^{6-}$ = 6 Na^+ + P^{6-} $\qquad c_P = \frac{m_{Na_6P}}{M} = \frac{75,6}{63000} = 0.0012$ mol dm^{-3}

NaCl = Na^+ + Cl^- $\qquad c_N = 0.18$ mol dm^{-3}

x mol dm^{-3} NaCl pass from the left compartment to the right one

Balance	at the beginning		in equilibrium	
	Left	Right	Left	Right
P^{6-}	c_P	0	c_P	0
Na^+	6 c_P + c_N	0	6 c_P + c_N − x	x
Cl^-	c_N	0	c_N − x	x

Donnan equilibrium condition for NaCl:

$$(c_{Na^+})_{Left} \cdot (c_{Cl^-})_{Left} = (c_{Na^+})_{Right} \cdot (c_{Cl^-})_{Right}$$

$$(6\ c_P + c_N - x) \cdot (c_N - x) = x \cdot x$$

$$(6\ c_P + c_N) \cdot c_N - x \cdot c_N\ (6\ c_P + c_N) \cdot x + x^2 = x^2$$

$$x = \frac{c_N \cdot (6\ c_P + c_N)}{6\ c_P + 2\ c_N} = \frac{0.18 \cdot (6 \cdot 0.0012 + 0.18)}{6 \cdot 0.0012 + 2 \cdot 0.18} = 0.091765 \text{ mol dm}^{-3}$$

$(c_{Na^+})_{Left}$ = 6 c_P + c_N − x = 6·0.0012 + 0.18 − 0.091765 = 0.095435 mol dm^{-3}

$(c_{Cl^-})_{Left}$ = c_N − x = 0.18 − 0.091765 = 0.088235 mol dm^{-3}

$(c_{Na^+})_{Right} = (c_{Cl^-})_{Right}$ = x = 0.0917647 mol dm^{-3}