

Problem 13-27 Osmotic pressure of non-ideal solutions

Concentration dependence of osmotic pressure of a non-uniform high-molecular substance, derived from experimental data, has the form

$$\pi = 10.92 \cdot c_w + 0.94 \cdot c_w^2$$

where π is the osmotic pressure (Pa), c_w mass concentration of the solution (kg m^{-3}).

(a) Calculate the molar mass of the studied polymer.

(b) How high will climb the solution level in the osmometric tube if the solution concentration is 0.04 mol m^{-3} ?

[(a) $M = 240.7 \text{ kg mol}^{-1}$, (b) $h = 2 \text{ cm}$]

Solution:

$$(a) \frac{\pi}{c_w} = \frac{RT}{\bar{M}_N} + RT \cdot B \cdot c_w$$

(Osmotic pressure is a colligative property \Rightarrow molar mass obtained from osmotic measurements represents a number average ($\bar{M}_N = \sum x_i M_i$))

$$\frac{RT}{\bar{M}_N} = 10.92 \text{ Pa m}^3 \text{ kg}^{-1}$$

$$\bar{M}_N = \frac{8.314 \cdot (43 + 273.15)}{10.92} = 240.7 \text{ kg mol}^{-1}$$

$$\bar{M}_N = 240.7 \text{ kg mol}^{-1} \quad \left[\bar{M}_N \right] = \left[\frac{(\text{J} \cdot \text{mol}^{-1} \text{ K}^{-1}) \cdot \text{K}}{\text{Pa m}^3 \text{ kg}^{-1}} = \frac{(\text{N} \cdot \text{m} \cdot \text{mol}^{-1} \text{ K}^{-1}) \cdot \text{K}}{\text{N} \cdot \text{m}^{-2} \cdot \text{m}^3 \text{ kg}^{-1}} = \text{kg mol}^{-1} \right]$$

(b) $c = 0.04 \text{ mol m}^{-3}$

$$c_w = c \cdot M = 0.04 \cdot 240.7 [(\text{mol m}^{-3}) \cdot (\text{kg mol}^{-1})] = 9.628 \text{ kg m}^{-3}$$

$$\pi = 10.92 \cdot 9.628 + 0.94 \cdot 9.628^2 = 192.274 \text{ Pa}$$

$$\pi = h \rho g$$

$$h = \frac{\pi}{\rho \cdot g} = \frac{192.274}{979.5 \cdot 9.81} = 0.02001 \text{ m} = 2 \text{ cm}$$