14. CURVED INTERFACES

$$p_\mathrm{i} - p_\mathrm{e} = \frac{2\gamma}{r}$$

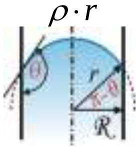
$$h = \frac{2\,\gamma_{\ell \mathrm{g}} \cos\theta}{\rho^{(\ell)}\, g\, \mathcal{R}}$$

convex surface

concave surface

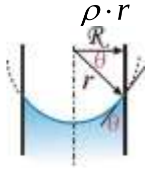
$$\boldsymbol{R}T \ln \frac{p_r^\mathrm{s}}{p_\infty^\mathrm{s}} = \frac{V_\mathrm{m}^\ell \cdot 2 \cdot \gamma}{r} = \frac{M \cdot 2 \cdot \gamma}{\rho \cdot r}$$

$$r = \frac{\mathcal{R}}{\cos\theta}$$



$$\boldsymbol{R}T \ln \frac{p_r^\mathrm{s}}{p_\infty^\mathrm{s}} = -\frac{V_\mathrm{m}^\ell \cdot 2 \cdot \gamma}{r} = -\frac{M \cdot 2 \cdot \gamma}{\rho \cdot r}$$

$$r = \frac{\mathcal{R}}{\cos(\pi - \theta)}$$



$$\boldsymbol{R}T \ln \frac{(c_i)_r}{(c_i)_\infty} = \frac{2\,\gamma \cdot V_\mathrm{m}^{s,\ell}}{r}$$

Equilibrium conditions on curved interfaces – Capillary elevation and depression

Problem 14-01 Pressure difference on curved phase boundary - Laplace-Young equation

Calculate the pressure inside the drop of a liquid of density with radius of 6 μm floating in the air (temperature 35 °C, pressure 102.1 kPa). The liquid density is 0.93 g cm^{-3} and surface tension 42 mN m^{-1}.

[$p^{(\ell)}$ = 116.1 kPa]

Problem 14-02 Capillary elevation

The stems of roses growing in the garden are about 50 cm in length. The cell walls are interwoven by a network of small canals (capillaries) enabling the water transport in plants. You can assume that water completely wets the walls of these capillaries and the average *diameter* of capillaries is 80 μm. Estimate if due to the capillary elevation can water climb up to the bloom. Surface tension of water is 72.6 mN m^{-1} and its density 0.998 g cm^{-3}.

[h = 0.37 m – water does not climb up to the blossom]

Problem 14-03 Capillary elevation

The surface tension of a certain liquid was determined by the capillary rise measurement. At the temperature of 25 °C the liquid level in the capillary of inner diameter 0.2 mm climbed up to the height of 9 cm above the planar liquid level in the wide vessel. The measured liquid completely wets the material of the capillary and its density is 1.076 g cm^{-3}. Calculate the surface tension of the liquid.

[γ = 47.5 mN m^{-1}]

Problem 14-04 Capillary elevation

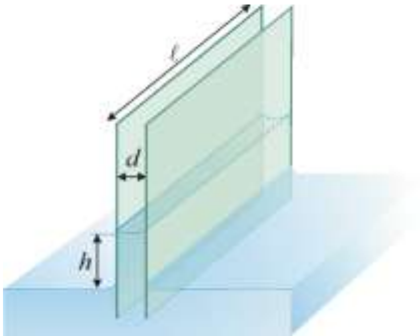
The wall of your cattle shed became wet up to the height of 1.5 m. Assuming the complete wetting, estimate an effective diameter of the pore in the wall. For the surface tension take the value of 71 mN m^{-1}, for density 1 g cm^{-3}. What pressure you should apply to prevent water from entering the wall?

[d = 19.3 μm; Δp = 14.715 kPa]

Problem 14-05 Capillary elevation between planar surfaces

Calculate the elevation h between two plane-parallel plates separated by a distance d = 0,6 mm, partially immersed into the liquid (see the attached illustration). The surface tension of the liquid is 45.6 mN m^{-1} and its density 0.886 g cm^{-3}. Suppose that the plates are so large, that the end effects are negligible and contact angle is zero.

[h = 17.5 mm]



Problem 14-06 Contact angle from capillary elevation

To determine the contact angle between glass and a liquid with surface tension of 18 mN m^{-1} and density ρ = 1.056 g cm^{-3}, the capillary rise method was used: at the temperature of 25 °C the measured liquid exhibited in a capillary of inner diameter 0.05 mm the capillary elevation 93 mm. Calculate the contact angle.

[θ = 48°]

Problem 14-07 Capillary elevation and depression, contact angle

The liquid A with the surface tension γ_A = 28 mN m^{-1} and density ρ_A = 1.16 g cm^{-3}, climbs in a glass capillary of the inner diameter of 0.04 mm up to the height of 92 mm above the level of the liquid in a broad tube.

(a) Calculate the contact angle of the liquid A on glass.

(b) A small amount of another liquid, B, with the same surface tension and density, forms on a flat glass plate a drop with the contact angle θ_B = 100.4°. How will this liquid behave in a vertical glass capillary – will it exhibit an elevation or a depression? Calculate it.

[(a) θ_A = 68°; (b) deprese h_B = –44 mm]

Vapour pressure above the curved interface –Kelvin equation

Problem 14-08 Kelvin equation – vapour pressure above a drop

Saturated vapour pressure above a liquid (M = 183 g mol^{-1}) at the temperature of 33 ºC is 5.2 kPa. Its surface tension is 68 mN m^{-1} and density 0.88 g cm^{-3}. What is the vapour pressure above a drop with diameter of 0.3 μm?

[p_r^s = 5.6 kPa]

Problem 14-09 Kelvin equation – vapour pressure above a drop

A liquid with molar mass $M = 260$ g mol^{-1}, density 0.855 g cm^{-3}, and surface tension 35 mN m^{-1}, was dispersed into air in the form of small droplets. Calculate the size of drops if you know that the vapour pressure above the drops is 16.8 kPa, whereas the equilibrium vapour pressure above this liquid with the planar surface has the value of 16.54 kPa, both at the temperature of 31 °C.

[$r = 0.54$ μm]

Problem 14-10 Kelvin equation – vapour pressure above a drop

What is the vapour pressure above the liquid drops of diameter of 18 nm at the temperature of 35 °C? The density of the liquid at this temperature is 0.022 mol cm^{-3}, its surface tension 43.5 mN m^{-1}, normal boiling point 78 °C, and the vaporization enthalpy has the value of 37.2 kJ mol^{-1}. You can suppose that the vaporization enthalpy is constant and that the temperature dependence of the vapour pressure is adequately described by Clausius-Clapeyron equation.

[p_r^s = 20.315 kPa]

Problem 14-11 Kelvin equation – concave surface

The surface tension of nitrobenzene is 39 mN m^{-1}, its molar volume 102.2 cm^3 mol^{-1}, and the equilibrium vapour pressure at the temperature of 303 K has the value p_∞^s = 2.271 kPa. At this temperature calculate the vapour pressure above the meniscus of nitrobenzene

(a) in a thin capillary of diameter 0.46 μm, which is completely wetted by nitrobenzene ($\theta = 0°$),

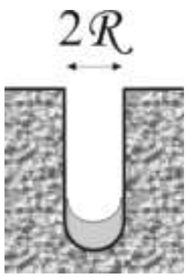
(b) in a capillary of the same diameter but made from a material, which is not completely wetted by nitrobenzene ($\theta = 55°$).

[(a) p_r^s = 2.24 kPa, (b) p_r^s = 2.253 kPa]

Problem 14-12 Kelvin equation – concave surface

Equilibrium vapour pressure of a liquid at the temperature of 40 °C is 4.2 kPa. Three moles of this liquid occupy at 40 °C the volume of 540 cm^3 and its surface tension is 48 mN m^{-1}. Calculate the pressure at which will condense the vapour of the studied substance on a porous solid with cylindrical pores (see attached picture) of diameter $6{\cdot}10^{-8}$ m. Assume complete wetting.

[p_r = 3.366 kPa]



Influence of curvature on solubility

Problem 14-13 Influence of the surface curvature on solubility

At the temperature of 322 K a solid substance in the form of large crystals dissolves to form a saturated solution of concentration 5.2 wt. %. If we will dissolve the same substance at the same temperature in the same solvent ($\rho^{(\ell)}$ = 0.823 g cm^{-3}), but in the form of small approximately spherical particles, the solubility will change. The interfacial tension solid/saturated solution is 62 mN m^{-1}, the studied substance does not dissociate on dissolving and its molar mass is M = 136 g mol^{-1}. Specific area of the small particles is 4.22 m^2/g. Calculate the percentage change in solubility.

[100 $(c_r - c_\infty)/c_\infty$ = 0.89 % (c_r = 5.2463 wt. %)]

Problem 14-14 Kelvin equation – concave surface

The equilibrium solubility of an organic liquid in water at 30 °C is 1.7 mmol dm^{-3}. The solute does not dissociate on dissolving. The solubility will be higher if the organic liquid will be dispersed in water in the form of tiny droplets. What should be the diameter of the droplets to increase the solubility at least by 1.5 %? The density of the organic liquid is ρ = 0.957 g cm^{-3}, its molar mass 118 g mol^{-1}, and interfacial tension between aqueous and organic phases is 35 mN m^{-1}.

[d =0.46 μm]