Problem 14-10 Kelvin equation – vapour pressure above a drop

What is the vapour pressure above the liquid drops of diameter of 18 nm at the temperature of 35 °C? The density of the liquid at this temperature is 0.022 mol cm^{-3}, its surface tension 43.5 mN m^{-1}, normal boiling point 78 °C, and the vaporization enthalpy has the value of 37.2 kJ mol^{-1}. You can suppose that the vaporization enthalpy is constant and that the temperature dependence of the vapour pressure is adequately described by Clausius-Clapeyron equation.

[p_r^{s} = 20.315 kPa]

Solution:

ρ_{m} = 0.022 mol cm^{-3} = 2.2·10^{4} mol m^{-3}

$V_m = 1/\rho_m$ = 1/2.2·10^{4} m^{3} mol^{-1}

T_{nbp} = 78 + 283.15 = 351.15 K

T = 35 + 283.15 = 308.15 K

d = 18·10^{-9} m

r = 9·10^{-9} m

$\Delta_{\mathrm{vap}}H_{\mathrm{m}}$ = 37.2 kJ mol^{-1}

$$\ln\frac{p_r^{\mathrm{s}}}{p_\infty^{\mathrm{s}}} = \frac{2\cdot\gamma\cdot V_m}{RT\cdot r} = \frac{2\cdot\gamma}{RT\cdot r\cdot\rho_m} = \frac{2\cdot 0.0435}{8.314\cdot 308.15\cdot 9\cdot 10^{-9}\cdot 2.2\cdot 10^{4}} = 0.171507$$

$p_r^{\mathrm{s}}(308.15\ \mathrm{K}) = 1.18657 \cdot p_\infty^{\mathrm{s}}(308.15\ \mathrm{K})$

Calculation of the vapour pressure above the curved interface:

$$\ln\frac{p_\infty^{\mathrm{s}}(308.15)}{101.325} = \frac{\Delta_{\mathrm{výp}}H_{\mathrm{m}}}{R}\left(\frac{1}{T_{\mathrm{nbv}}} - \frac{1}{T}\right) = \frac{37.2\cdot 10^{3}}{8.314}\left(\frac{1}{351.15} - \frac{1}{308.15}\right) = -1.77806$$

$p_\infty^{\mathrm{s}}(308.15\ \mathrm{K}) = 0.16897 \cdot 101.325 = 17.12043\ \mathrm{kPa}$

$p_r^{\mathrm{s}}(308.15\ \mathrm{K}) = 1.18657 \cdot 17.12043 = 20.315\ \mathrm{kPa}$