Problem 16-01 Gibbs adsorption isotherm

The concentration dependence of the surface tension of diluted solutions of a surfactant at the temperature of 25 °C was expressed by the following equation:

$$\gamma = 7.2\cdot10^{-2} - 6.445\cdot10^{-3}\cdot c_2 + 1.14\cdot10^{-3}\cdot c_2^{\,2}$$

c_2 is the concentration of the solute (mol dm^{-3}) and γ surface tension (N m^{-1}). Find an equation describing the concentration dependence of the relative Gibbs adsorption, $\Gamma_{2,1}$.

$[\Gamma_{2,1} = 2.6 \cdot 10^{-6} \cdot c_2 - 9.2 \cdot 10^{-7} \cdot c_2^{\,2}]$

Solution:

$$\Gamma_{2,1} = -\frac{1}{\boldsymbol{R}T}\left(\frac{\mathrm{d}\gamma}{\mathrm{d}\ln c_2}\right)_{T,\,p} = -\frac{c_2}{\boldsymbol{R}T}\left(\frac{\mathrm{d}\gamma}{\mathrm{d}c_2}\right)_{T,\,p}$$

$T = 298.15$ K

$$\left(\frac{\mathrm{d}\gamma}{\mathrm{d}c_2}\right)_{T,\,p} = -6.445\cdot10^{-3} + 2\cdot1.14\cdot10^{-3}\cdot c_2 = -6.4\cdot10^{-3} + 2.28\cdot10^{-3}\cdot c_2$$

$$\Gamma_{2,1} = -\frac{6.445\cdot10^{-3}}{8.314\cdot298.15}\cdot c_2 - \frac{2.28\cdot10^{-3}}{8.314\cdot298.15}\cdot c_2^2$$

$$\Gamma_{2,1} = 2.6 \cdot 10^{-6} \cdot c_2 - 9.2 \cdot 10^{-7} \cdot c_2^2$$