**Problem 4-03** Constants of Michaelis-Menten equation from differential data; integrated equation ($c_{S0} >> K_M$)

Polarimetric study of enzymatic hydrolysis yielded the dependence of the initial reaction rate on the substrate concentration. Some of the gained values are given in the following table:

| | | |
|---|---|---|
| $c_S$ /(mol dm$^{-3}$) | 0.062 | 1.82 |
| $10^6\ v_0$ / (mol dm$^{-3}$ s$^{-1}$) | 4.96 | 5.48 |

(a) Calculate the constants of Michaelis-Menten equation $K_M$ and $v_{max}$.
(b) What will be the conversion of optically active substrate after 6 hours from the instant when the enzyme was added to the substrate solution, the concentration of which was 2.2 mol dm$^{-3}$?

[(a) $K_M$ = 6.754· 10$^{-3}$ mol dm$^{-3}$ , $v_{max}$ = 5.5 · 10$^{-6}$ mol dm$^{-3}$ s$^{-1}$; (b) $\alpha$ = 0.054]

Solution:

$$v = -\frac{dc_S}{d\tau} = \frac{dc_P}{d\tau} = \frac{v_{max} \cdot c_S}{K_M + c_S} \qquad \text{kde} \quad v_{max} = k_2 \cdot c_{E0}$$

(a) Linearization (Lineweaver and Burke):

$$\frac{1}{v} = \frac{1}{v_{max}} + \frac{K_M}{v_{max}} \cdot \frac{1}{c_S}$$

$$\frac{1}{4.96 \cdot 10^{-6}} = \frac{1}{v_{max}} + \frac{K_M}{v_{max}} \cdot \frac{1}{0.062}$$

$$\frac{1}{5.48 \cdot 10^{-6}} = \frac{1}{v_{max}} + \frac{K_M}{v_{max}} \cdot \frac{1}{1.82}$$

$$\frac{K_M}{v_{max}} = \frac{\frac{1}{4.96 \cdot 10^{-6}} - \frac{1}{5.48 \cdot 10^{-6}}}{\frac{1}{0.062} - \frac{1}{1.82}} = 1227.963\,\text{s}$$

$$\frac{1}{v_{max}} = \frac{1}{4.96 \cdot 10^{-6}} - 1228 \cdot \frac{1}{0.062} = 181806.45 \quad , v_{max} = 5.5 \cdot 10^{-6} \text{ mol dm}^{-3}\text{ s}^{-1}$$

$K_M$ = 1228 · 5.5 ·10$^{-6}$ = 6.754· 10$^{-3}$ mol dm$^{-3}$

(b) $c_{S0}$ = 2.2 mol dm$^{-3}$ >> $K_M$ = 3.679 · 10$^{-3}$ mol dm$^{-3}$

$$-\frac{dc_S}{d\tau} = \frac{v_{max} \cdot c_S}{c_S} = v_{max}$$

Balance: $c_S = c_{S0} - c_{S0} \cdot \alpha$ , $dc_S = -c_{S0} \cdot d\alpha$

$$+\frac{c_{S0}\, d\alpha}{d\tau} = v_{max}$$

$$\alpha = \frac{v_{max}}{c_{S0}} \cdot \tau = \frac{5.5 \cdot 10^{-6}}{2.2} \cdot 6 \cdot 3600 = 0.054$$