Problem 4-04 Constants of Michaelis-Menten equation from integral data

Determine the kinetic parameters K_M and v_{max} describing the action of chymotrypsin from bovine pancreas on N-acetyl-L-tyrosine ethyl ester (substrate S) using these data measured at 25°C and pH = 7.8:

τ /min	0	17.2	32.5
c_S / mol dm^{-3}	$2.0 \cdot 10^{-4}$	$1.4 \cdot 10^{-4}$	$1.0 \cdot 10^{-4}$

[$K_M = 6.966 \cdot 10^{-4}$ mol dm^{-3}, $v_{max} = 1.7933 \cdot 10^{-5}$ mol dm^{-3} s^{-1}]

Solution:

$$v = \frac{dc_P}{d\tau} = -\frac{dc_S}{d\tau} = \frac{v_{max} \cdot c_S}{K_M + c_S}$$

$$-\frac{dc_S}{d\tau} = \frac{v_{max} \cdot c_S}{K_M + c_S}$$

$$-\left(\frac{K_M}{c_S} + 1\right) dc_S = v_{max} \cdot d\tau$$

$$-K_M \cdot \ln\frac{c_S}{c_{S0}} - (c_S - c_{S0}) = v_{max} \cdot \tau$$

$$K_M \cdot \ln\frac{c_{S0}}{c_S} + (c_{S0} - c_S) = v_{max} \cdot \tau$$

$$K_M \cdot \ln\frac{2.0 \cdot 10^{-4}}{1.4 \cdot 10^{-4}} + (2.0 - 1.4) \cdot 10^{-4} = v_{max} \cdot 17.2$$

$$K_M \cdot \ln\frac{2.0 \cdot 10^{-4}}{1.0 \cdot 10^{-4}} + (2.0 - 1.0) \cdot 10^{-4} = v_{max} \cdot 32.5$$

$$K_M \cdot 0.0207369 + 3.488372 \cdot 10^{-6} = v_{max} = K_M \cdot 0.0213276 + 3.07692 \cdot 10^{-6}$$

$$K_M = \frac{3.488372 \cdot 10^{-6} - 3.07692 \cdot 10^{-6}}{5.9069 \cdot 10^{-4}} = 6.9656 \cdot 10^{-4} \text{ mol dm}^{-3}$$

$$v_{max} = 6.9656 \cdot 10^{-4} \cdot 0.0213276 + 3.07692 \cdot 10^{-6}$$

$$v_{max} = 1.7933 \cdot 10^{-5} \text{ mol dm}^{-3} \text{ min}^{-1}$$