Problem 4-05 Integrated Michaelis-Menten equation; $c_{S0} \approx K_M$; the amount of enzyme

Calculate what amount of enzyme (μg) should be added to 15 cm^3 of the solution, containing 0.168 mmol of substrate S, in order that the conversion of 35 % of initially present substrate was reached in 40 seconds from the beginning of the reaction. The molar mass of the enzyme is 38 kg mol^{-1} and its catalytic activity is $k_2 = 8 \cdot 10^5$ μmol of substrate the enzyme converts per 1 μmol of the enzyme per one second. Michaelis constant for this system has the value $K_M = 8.13$ mmol dm^{-3}.

[$m_{E0} = 1.322 \cdot 10^{-10}$ kg =0.132 μg]

Solution:

$c_{S0} \approx K_M$

Data:

$c_{E0} = ?$ $\quad V = 15\ cm^3$, $\quad \alpha = 0.35$, $\quad \tau = 40$ s

$k_2 = 8 \cdot 10^5\ mol_{substrate}\ (mol_{enzym})^{-1}\ s^{-1}$

$$c_{S0} = \frac{1.68 \cdot 10^{-4}}{15 \cdot 10^{-3}} = 0.0112 \text{ mol dm}^{-3}$$

$$K_M = 8.13 \cdot 10^{-3} \text{ mol dm}^{-3}$$

} the same order of magnitude

$$v_0 = \frac{k_2 \cdot c_{E0} \cdot c_S}{K_M + c_S}$$

$$c_S = c_{S0} - c_{S0} \cdot \alpha \quad , \quad dc_S = -c_{S0} \cdot d\alpha$$

$$c_{S0} \cdot \frac{d\alpha}{d\tau} = k_2 \cdot c_{E0} \frac{c_{S0}(1-\alpha)}{K_M + c_{S0}(1-\alpha)}$$

$$k_2 \cdot c_{E0} \cdot d\tau = \frac{K_M + c_{S0} \cdot (1-\alpha)}{(1-\alpha)} d\alpha = \left(\frac{K_M}{(1-\alpha)} + c_{S0} \right) d\alpha$$

$$k_2 \cdot c_{E0} \cdot \tau = \int_0^x \left(\frac{K_M}{(1-\alpha)} + c_{S0} \right) d\alpha = -K_M \cdot \ln(1-\alpha) + c_{S0} \cdot \alpha$$

$$c_{E0} = \frac{-K_M \cdot \ln(1-\alpha) + c_{S0} \cdot \alpha}{k_2 \cdot \tau} = \frac{-8.13 \cdot 10^{-3} \cdot \ln(1-0.35) + 1.12 \cdot 10^{-2} \cdot 0.35}{8 \cdot 10^5 \cdot 40}$$

$$= 2.3195 \cdot 10^{-10}\ mol_{enzyme}\ dm^{-3}$$

$$m_{E0} = n_{E0} \cdot M_E = c_{E0} \cdot V \cdot M_E = 2.3195 \cdot 10^{-10} \cdot 15 \cdot 10^{-3} \cdot 38$$

$$[(mol_{enzyme}\ dm^{-3}) \cdot (dm^3) \cdot (kg\ mol^{-1})]$$

$m_{E0} = 1.322 \cdot 10^{-10}$ kg = 0.132 μg