Problem 9-05 Heterogeneous ionic equilibria – precipitation from solution

Acid mine water contains 0.001 mol Fe^{3+}/dm^3. Determine the pH at which begins the precipitation of $Fe(OH)_3$. The solubility product of $Fe(OH)_3$ at the temperature of 18 °C is $3.8 \cdot 10^{-38}$, the ionic product of water $5.78 \cdot 10^{-15}$ (both for the standard state of infinite dilution, $c^{st} = 1$ mol dm^{-3}). Assume that activities can be replaced by relative concentrations.

[pH = 2.8]

Solution:

$Fe(OH)_3\ (s) = Fe^{3+} + 3\ OH^-$

$c_{Fe^{3+}} = c\ ,\ c_{OH^-} = 3\ c$

$$K_S = a_{Fe^{3+}} \cdot a_{OH^-}^3 = \gamma_+ \cdot \frac{c_{Fe^{3+}}}{c^{st}} \cdot \left(\gamma_- \cdot \frac{c_{OH^-}}{c^{st}} \right)^3 = \frac{\gamma_\pm^4}{c^{st}} \cdot c_{Fe^{3+}} \cdot c_{OH^-}^3$$

$a_i = c_i / c^{st}$ $\quad (\gamma_\pm = 1)\ ,\quad c^{st} = 1$ mol dm^{-3}

$$c_{OH^-} = \left(\frac{K_S}{c_{Fe^{3+}}} \right)^{1/3} = \left(\frac{3.8 \cdot 10^{-38}}{0.001} \right)^{1/3} = 3.362 \cdot 10^{-12} \text{ mol dm}^{-3}$$

$$c_{H^+} = \frac{K_V}{c_{OH^-}} = \frac{5.78 \cdot 10^{-15}}{3.362 \cdot 10^{-12}} = 1.719 \cdot 10^{-3} \text{ mol dm}^{-3}$$

$pH = -\log (1.719 \cdot 10^{-3}) = 2.7647$

pH = 2.8