Difference Approximations to the derivative

The Difference Approximations for first derivative are in the form

The formula a)

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h} + R(f)\,,$$

where $R(f) = \frac{1}{2}hf''(\xi) = \mathcal{O}(h)$ and $\xi \in (x_0, x_0+h)$.

The formula b)

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h} + R(f)\,,$$

where $R(f) = \frac{1}{6}h^2 f'''(\xi) = \mathcal{O}(h^2)$ and $\xi \in (x_0-h, x_0+h)$.

The Difference Approximation for second derivative is in the form

The formula for second derivative

$$f''(x_0) = \frac{f(x_0-h) - 2f(x_0) + f(x_0+h)}{h^2} + R(f)\,,$$

where $R(f) = \frac{1}{12}h^2 f^{(4)}(\xi) = \mathcal{O}(h^2)$ and $\xi \in (x_0-h, x_0+h)$.

Exercises:
Write a computer program to test the approximation to the first derivative and to the second derivative in the point $x_0 = 1$, for $h = 10^{-1}, 10^{-2}, 10^{-4}$

1. $f(x) = \mathrm{e}^{-x}$
2. $f(x) = \ln(x)$
3. $\cos \pi x$
4. $\sqrt{1+x}$
5. $\ln(\mathrm{e}^{\sqrt{x^2+1}} \sin(\pi x) + \tan(\pi x))$