# Lagrange Interpolating Polynomial

The Lagrange interpolating polynomial is the polynomial of degree $n$ that passes through the $n+1$ points $(x_0, y_0 = f(x_0)), \ldots, (x_n, y_n = f(x_n))$ and is given by

$$L_n(x) = \sum_{i=0}^{n} f(x_i) \, \frac{\omega_n(x)}{(x - x_i)\, \omega_n'(x_i)} \,, \quad \text{where } \omega_n(x) = (x - x_0)(x - x_1) \ldots (x - x_n)$$

or

$$L_n(x) = \sum_{i=0}^{n} f(x_i) \, l_i(x) \,, \quad \text{where } l_i(x) = \prod_{j=0, j \neq i}^{n} \frac{x - x_j}{x_i - x_j}$$

The family of functions $l_i(x)$ are polynomials and have the property that

$$l_i(x_j) = \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \,. \end{cases}$$

**Interpolation error**

Let $f \in C^{n+1}([a, b])$ and $x_i \in [a, b]$ for $0 \leq i \leq n$. Then, for each $x \in [a, b]$, there is a $\xi_x \in [a, b]$ such that

$$f(x) - L_n(x) = \frac{\omega_n(x)}{(n+1)!} f^{(n+1)}(\xi_x) \,.$$

**Exercises:**

1. $f(x) = \sqrt{(x)} \,,\ x \in [0, 1]$, 4-degree, 8-degree, 12-degree
2. $f(x) = \ln(x) \,,\ x \in [1, 2]$, 4-degree, 8-degree, 12-degree
3. $f(x) = \log_2(x) \,,\ x \in [1, 2]$, 4-degree, 8-degree, 12-degree
4. $f(x) = \sin(\pi x) \,,\ x \in [-1, 1]$, 4-degree, 8-degree, 12-degree