

## Program na řešení diferenciální rovnice 2. řádu metodou sítí

```
function [x, y] = ODEsiteNewton(n, f, df2, df3, a, b, alpha1,alpha2, beta1,beta2, gamma1,
gamma2, y_init, N_eps, N_maxiter)
% Funkce ODEsiteNewton resi obycejnou diferencialni
% rovnici druheho radu y''=f(x,y,y') na intervalu [a, b]
% metodou siti. Nelinearni algebraicke rovnice se resi
% Newtonovou metodou
%
% f .. funkce prave strany
% df2 = df/dy .. derivace f podle y
% df3 = df/dy'.. derivace f podle y'
%
% okrajove podminky:
% alpha1*y(a) + beta1*y'(a) = gamma1
% alpha2*y(b) + beta2*y'(b) = gamma2
%
% y_init .. pocatecni nastrel reseni pro Newtonovu metodu
% N_eps .. stopping kriterium pro Newtonovu metodu
% N_maxiter .. maximalni pocet Newtonovych iteraci

h = (b-a)/n;
y_old = [ ];
y_old = y_init;
y_old(1) = (gamma1 - 4*beta1/(2*h)*y_old(2) + beta1/(2*h)*y_old(3))/(alpha1 - 3*beta1/(2*h));
y_old(n+1) = (gamma2 + 4*beta2/(2*h)*y_old(n) - beta2/(2*h)*y_old(n-1))/(alpha2 + 3*beta2/(2*h));

for i = 1:n+1
    x(i) = a + (i-1)*h;
end
y = [ ];

error = N_eps+1;
k = 0;
while error >= N_eps && k <= N_maxiter
    F(1) = - ( 1/(h*h)*(y_old(1) - 2*y_old(2) + y_old(3)) - f(x(2),y_old(2),(y_old(3) - y_old
(1))/(2*h)));
    a(1) = 0;
    b(1) = - 2/(h*h) - df2(x(2),y_old(2),(y_old(3) - y_old(1))/(2*h)) - 1/(h*h)*(4*beta1/
(2*h))/(alpha1 - 3*beta1/(2*h)) - df3(x(2),y_old(2),(y_old(3) - y_old(1))/(2*h))* 1/(2*h)*
(4*beta1/(2*h))/(alpha1 - 3*beta1/(2*h));
    c(1) = 1/(h*h) - 1/(2*h) * df3(x(2),y_old(2),(y_old(3) - y_old(1))/(2*h)) + 1/(h*h)*
(beta1/(2*h))/(alpha1 - 3*beta1/(2*h)) - df3(x(2),y_old(2),(y_old(3) - y_old(1))/(2*h))* 1/(2*h)
*( beta1/(2*h))/(alpha1 - 3*beta1/(2*h));

    for i = 2:n-2
        F(i) = -( 1/(h*h)*(y_old(i) - 2*y_old(i+1) + y_old(i+2)) - f(x(i+1),y_old(i+1),(y_old(i+2)
- y_old(i))/(2*h)));
        a(i) = 1/(h*h) + 1/(2*h) * df3(x(i+1),y_old(i+1),(y_old(i+2) - y_old(i))/(2*h));
        b(i) = -2/(h*h) - df2(x(i+1),y_old(i+1),(y_old(i+2) - y_old(i))/(2*h));
        c(i) = 1/(h*h) - 1/(2*h) * df3(x(i+1),y_old(i+1),(y_old(i+2) - y_old(i))/(2*h));
    end

    F(n-1) = -(1/(h*h)*(y_old(n-1) - 2*y_old(n) + y_old(n+1)) - f(x(n),y_old(n),(y_old(n+1) - y_old
(n-1))/(2*h)));
    a(n-1) = 1/(h*h) + 1/(2*h) * df3(x(n),y_old(n),(y_old(n+1) - y_old(n-1))/(2*h)) - 1/(h*h)
*(beta2/(2*h))/(alpha2 + 3*beta2/(2*h)) + df3(x(n),y_old(n),(y_old(n+1) - y_old(n-1))/(2*h))*
```

```

1/(2*h)*( beta2/(2*h))/(alpha2 + 3*beta2/(2*h));
b(n-1) = - 2/(h*h) - df2(x(n),y_old(n),(y_old(n+1) - y_old(n-1))/(2*h)) + 1/(h*h)*(4*beta2/
(2*h))/(alpha2 + 3*beta2/(2*h)) - df3(x(n),y_old(n),(y_old(n+1) - y_old(n-1))/(2*h))* 1/(2*h)
*(4*beta2/(2*h))/(alpha2 + 3*beta2/(2*h));
c(n-1) = 0;

% reseni soustavy s Jacobianem (tridiagonalni matice)
delta_y = TriDiagonalSolve(a,b,c,F);

% vypocet dalsi Newtonovy iterace
y_new = [ ];
for i = 2:n
    y_new(i) = y_old(i) + delta_y(i-1);
end
y_new(1) = (gamma1 - 4*beta1/(2*h)*y_new(2) + beta1/(2*h)*y_new(3))/(alpha1 - 3*beta1/(2*h));
y_new(n+1) = (gamma2 + 4*beta2/(2*h)*y_new(n) - beta2/(2*h)*y_new(n-1))/(alpha2 + 3*beta2/(2*h));

% euklidovska norma dvou Newtonovych iteraci
error = norm(y_new - y_old);
k = k + 1;
fprintf(1,'k = %d, ',k);
fprintf(1,'error = %16.8e\n',error);
y_old = y_new;
end

if error < N_eps
    y = y_new;
    fprintf(' x y(x)\n');
    for i = 1:(n+1)
        fprintf('%5.3f, %10.6f\n',x(i),y(i));
    end
else
    fprintf('Metoda nekonverguje. Vycerpan maximalni pocet iteraci Newtonovy metody\n');
end

end

```

## Použití metody ODEsiteNewton na daný příklad

```
% y''= y, y(0)=1, y(1)=1
n = 20;
f = inline('y','x','y','dy');
df2 = inline('1','x','y','dy');
df3 = inline('0','x','y','dy');
a = 0; b = 1;
alpha1 = 1;
alpha2 = 1;
beta1 = 0;
beta2 = 0;
gamma1 = 1;
gamma2 = 1;
y_init = [];
for i = 2:n;
    y_init(i) = 0.5;
end
y_init(1) = 1;
y_init(n+1) = 1;

N_eps = 1e-6;
N_maxiter = 10;
[x, y] = ODEsiteNewton(n, f, df2, df3, a, b, alpha1,alpha2, beta1,beta2, gamma1,gamma2, y_init,
N_eps, N_maxiter);
if (~isempty(y))
    figure
    plot (x,y)
    title('Příklad 1')
    xlabel('x')
    ylabel('y(x)')
end
```

```
k = 1, error = 1.83744885e+00
k = 2, error = 1.53435875e-15
    x      y(x)
0.000, 1.000000
0.050, 0.978139
0.100, 0.958723
0.150, 0.941704
0.200, 0.927040
0.250, 0.914693
0.300, 0.904633
0.350, 0.896834
0.400, 0.891277
0.450, 0.887949
0.500, 0.886840
0.550, 0.887949
0.600, 0.891277
0.650, 0.896834
0.700, 0.904633
0.750, 0.914693
0.800, 0.927040
0.850, 0.941704
```

0.900, 0.958723  
0.950, 0.978139  
1.000, 1.000000

