

Rěšení parciální diferenciální rovnice parabolického typu

nelinearita je řešena metodou q u a s ilinearizac e

$$\frac{\partial}{\partial t} u = g_1(x, t) \frac{\partial^2}{\partial x^2} u + g_2(x, t) \frac{\partial}{\partial x} u + f(x, t, u), \quad x \in (a, b), t \geq 0$$

s okrajovými podmínkami :

$$\alpha_1 u(a, t) + \beta_1(t) \frac{\partial}{\partial x} u(a, t) = \gamma_1(t) \quad a \quad \alpha_2 u(b, t) + \beta_2(t) \frac{\partial}{\partial x} u(b, t) = \gamma_2(t)$$

a počáteční podmínkou : $u(x, 0) = \phi(x)$

n – počet bodů sítě v x, x ∈ (a, b)

m – počet bodů sítě v t

k – časový krok

```
PDEParabCN[n_, m_, k_, a_, b_, g_, e_, f_, a1_, a2_, b1_, b2_, g1_, g2_, phi_] := Module[{i, j, h, alpha, beta, d1, d2, d3, x1, u1, up, x, t1, F, m1, n1, u0, u, res, p, r, q, s, pom, k1, df},
{
df[x_, t_, u_] = D[f[x, t, u], u];
h = N[(b - a) / n];
m1 = m + 1;
n1 = n + 1;
k1 = k / 2;
alpha = N[k / (2 * h * h)];
beta = N[k / (4 * h)];
x = Table[a + (i - 1) h, {i, 1, n1}];
u0 = Table[phi[x[[i]]], {i, 1, n1}];
u = Table[0, {m1}, {n1}];
(* výpočet profilu u - 0 vrstva *)
For[i = 1, i <= n1, i++, u[[1, i]] = u0[[i]];
For[j = 1, j <= m, j++,
(* Napočítání diagonál matice a pravé strany *)
d1 = -Table[g[x[[i]], k * j] alpha - e[x[[i]], k * j] beta, {i, 2, n}];
d3 = -Table[g[x[[i]], k * j] alpha + e[x[[i]], k * j] beta, {i, 2, n}];
d2 = Table[1 + 2 * alpha * g[x[[i]], k * j] - k1 * df[x[[i]], (j) k, u[[j, i]]], {i, 2, n}];
F = Table[(g[x[[i]], k * (j - 1)] alpha - e[x[[i]], k * (j - 1)] beta) u[[j, i - 1]] + (1 - 2 * alpha * g[x[[i]], k * (j - 1)] - k1 * df[x[[i]], k * j, u[[j, i]]) * u[[j, i]] +
(g[x[[i]], k * (j - 1)] alpha + e[x[[i]], k * (j - 1)] beta) * u[[j, i + 1]] + k1 * (f[x[[i]], j k, u[[j, i]]] + f[x[[i]], (j - 1) k, u[[j, i]]]), {i, 2, n}];
(* dosazení okrajových podmínek do první a poslední rovnice*)
p = 1 / (alpha * 2 * h - 3 * beta[j k]);
r = beta[j k] * p;
pom = d1[[1]] * r;
d2[[1]] = d2[[1]] - 4 * pom;
d3[[1]] = d3[[1]] + pom;
F[[1]] = F[[1]] - 2 * h * g1[j k] * d1[[1]] * p;
q = 1 / (alpha * 2 * h + 3 * beta[j k]);
s = beta[j k] * q;
pom = d3[[n - 1]] * s;
d2[[n - 1]] = d2[[n - 1]] + 4 * pom;
d1[[n - 1]] = d1[[n - 1]] - pom;
F[[n - 1]] = F[[n - 1]] - 2 * h * g2[j k] * d3[[n - 1]] * q;
(* výpočet profilu u - j+1 vrstva *)
up = TriDiagonalSolve[d1, d2, d3, F];
For[i = 2, i <= n, i++, u[[j + 1, i]] = up[[i - 1]];
(* výpočet krajních hodnot j+1 profilu *)
u[[j + 1, 1]] = 2 * h * g1[j k] * p - 4 * r * u[[j + 1, 2]] + r * u[[j + 1, 3]];
u[[j + 1, n1]] = 2 * h * g2[j k] * q + 4 * s * u[[j + 1, n]] - s * u[[j + 1, n - 1]];
];
(*res=Table[{(i-1)*h,u[[j,i]],{j,1,m1},{i,1,n1}}];*)
u
}
]
```

Help

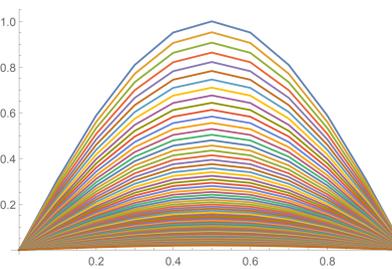
PDEParabCNC[n, m, k, a, b, g, e, f, alpha1, alpha2, beta1, beta2, gamma1, gamma2, phi] stejné jako PDEParabNC, nahraza první derivace v okrajových podmínkách je centrální.

```
PDEParabCNC[n_, m_, k_, a_, b_, g_, e_, f_, a1_, a2_, b1_, b2_, g1_, g2_, phi_] := Module[{i, j, h, alpha, beta, d1, d2, d3, x1, u1, up, x, t1, F, m1, n1, u0, u, res, p, q, k1, df, pom1, pom2},
{
df[x_, t_, u_] = D[f[x, t, u], u];
h = N[(b - a) / n];
m1 = m + 1;
n1 = n + 1;
k1 = k / 2;
alpha = N[k / (2 * h * h)];
beta = N[k / (4 * h)];
x = Table[a + (i - 1) h, {i, 1, n1}];
u0 = Table[phi[x[[i]]], {i, 1, n1}];
u = Table[0, {m1}, {n1}];
(* výpočet profilu u - 0 vrstva *)
For[i = 1, i <= n1, i++, u[[1, i]] = u0[[i]];
For[j = 1, j <= m, j++,
(* Napočítání diagonál matice a pravé strany *)
d1 = -Table[g[x[[i]], k * j] alpha - e[x[[i]], k * j] beta, {i, 1, n1}];
d3 = -Table[g[x[[i]], k * j] alpha + e[x[[i]], k * j] beta, {i, 1, n1}];
d2 = Table[1 + 2 * alpha * g[x[[i]], k * j] - k1 * df[x[[i]], (j) k, u[[j, i]]], {i, 1, n1}];
F = Table[(g[x[[i]], k * (j - 1)] alpha - e[x[[i]], k * (j - 1)] beta) u[[j, i - 1]] + (1 - 2 * alpha * g[x[[i]], k * (j - 1)] - k1 * df[x[[i]], k * j, u[[j, i]]) * u[[j, i]] +
(g[x[[i]], k * (j - 1)] alpha + e[x[[i]], k * (j - 1)] beta) * u[[j, i + 1]] + k1 * (f[x[[i]], j k, u[[j, i]]] + f[x[[i]], (j - 1) k, u[[j, i]]]), {i, 2, n}];
F = Prepend[F, (g[x[[1]], k * (j - 1)] alpha - e[x[[1]], k * (j - 1)] beta) pom1 + (1 - 2 * alpha * g[x[[1]], k * (j - 1)] - k1 * df[x[[1]], k * j, u[[j, 1]]) * u[[j, 1]] +
(g[x[[1]], k * (j - 1)] alpha + e[x[[1]], k * (j - 1)] beta) * u[[j, 2]] + k1 * (f[x[[1]], j k, u[[j, 1]]] + f[x[[1]], (j - 1) k, u[[j, 1]]));
F = Append[F, (g[x[[n1]], k * (j - 1)] alpha - e[x[[n1]], k * (j - 1)] beta) u[[j, n]] + (1 - 2 * alpha * g[x[[n1]], k * (j - 1)] - k1 * df[x[[n1]], k * j, u[[j, n1]]) * u[[j, n1]] +
(g[x[[n1]], k * (j - 1)] alpha + e[x[[n1]], k * (j - 1)] beta) * pom2 + k1 * (f[x[[n1]], j k, u[[j, n1]]] + f[x[[n1]], (j - 1) k, u[[j, n1]]));
(* dosazení okrajových podmínek do první a poslední rovnice*)
If[beta1[t] == 0,
{d2[[1]] = a1; d3[[1]] = 0; F[[1]] = gamma1[j k]},
{p = 2 * h / beta[j k];
d2[[1]] = d2[[1]] + a1 * d1[[1]] * p;
d3[[1]] = d3[[1]] + d1[[1]];
F[[1]] = F[[1]] + gamma1[j k] * d1[[1]] * p};
];
If[beta2[t] == 0,
{d2[[n1]] = a2; d1[[n1]] = 0; F[[n1]] = gamma2[j k]},
{q = 2 * h / beta[j k];
d2[[n1]] = d2[[n1]] - a2 * d3[[n1]] * q;
d1[[n1]] = d1[[n1]] + d3[[n1]];
F[[n1]] = F[[n1]] - gamma2[j k] * d3[[n1]] * q};
];
(* výpočet profilu u - j+1 vrstva *)
up = TriDiagonalSolve[d1, d2, d3, F];
For[i = 1, i <= n1, i++, u[[j + 1, i]] = up[[i]];
(* výpočet pomocných krajních hodnot j+1 profilu *)
If[beta1[t] == 0, pom1 = gamma1[j k] / a1, pom1 = (-gamma1[j k] + a1 * u[[j + 1, 1]]) * p + u[[j + 1, 2]];
If[beta2[t] == 0, pom2 = gamma2[j k] / a2, pom2 = (gamma2[j k] - a2 * u[[j + 1, n1]]) * q + u[[j + 1, n]];
];
(*res=Table[{(i-1)*h,u[[j,i]],{j,1,m1},{i,1,n1}}];*)
u
}
]
```

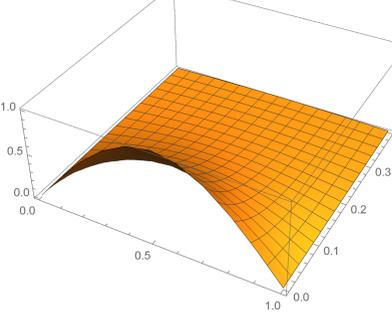
Příklad 1:

$$\frac{\partial}{\partial t} u = \frac{\partial^2}{\partial x^2} u \quad u(0,t)=0, u(1,t)=0, u(x,0)=\sin(x\pi)$$

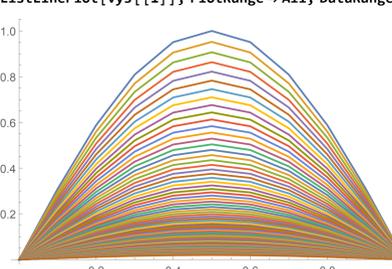
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phi[x_] = Sin[Pi x];
a1 = 1;
beta1[t_] = 0;
alpha2 = 1;
beta2[t_] = 0;
gamma1[t_] = 0;
gamma2[t_] = 0;
g[x_, t_] = 1;
e[x_, t_] = 0;
f[x_, t_, u_] = 0;
n = 10;
m = 80;
k = 0.005;
T = k * m;
vys = PDEParabCNC[n, m, k, 0.0, 1.0, g, e, f, alpha1, alpha2, beta1, beta2, gamma1, gamma2, phi];
ListLinePlot[vys[[1]], PlotRange -> All, DataRange -> {0, 1}]
```



```
ListPlot3D[vys[[1]], PlotRange -> All, DataRange -> {{0, 1}, {0, T}}]
```



```
vys = PDEParabCNC[n, m, k, 0.0, 1.0, g, e, f, alpha1, alpha2, beta1, beta2, gamma1, gamma2, phi];
ListLinePlot[vys[[1]], PlotRange -> All, DataRange -> {0, 1}]
```



```
ListPlot3D[vys[[1]], PlotRange -> All, DataRange -> {{0, 1}, {0, T}}]
```

