řešení parciální diferenciální rovnice metodou Crank – Nicolsonové

```
PDEParabCN := proc (n, m, k, a, b, g, e, f, alfa1, beta1, alfa2, beta2, gama1, gama2, phi)
    local h, n1, m1, u, i, j, alfa, beta, x, F, d1, d2, d3, p, q, r, s, pom, up, df, k1;
    with(linalg);
    h := (b-a)/n;
    n1 := n+1;
    m1 := m+1;
    k1 := k/2;
    alfa := evalf(k/(2*h*h));
    beta := evalf(k/(4*h));
    df := D[3](f);
    x := Vector(n1, i -> a+(i-1)*h);
    u := Matrix((m1, n1), 0);
    d1 := Vector(n-1, 0);
    d2 := Vector(n-1, 0);
    d3 := Vector(n-1, 0);
    F := Vector(n-1, 0);
    #profil u - 0 vrstva
    for i from 1 to n1 do
        u[1, i] := evalf(phi(x[i]));
    end do;
    #Výpočet diagonál v matici A a pravé strany, u(j+1)=F(j)+A*u(j)
    for j from 1 to m do
        for i from 1 to n-1 do
            d1[i] := -evalf(g(x[i+1], j*k)*alfa-e(x[i+1], j*k)*beta);
            d2[i] := evalf(1+2*alfa*g(x[i+1], j*k)-k1*df(x[i+1], j*k, u[j, i+1]));
            d3[i] := -evalf(g(x[i+1], j*k)*alfa+e(x[i+1], j*k)*beta);
            F[i] := evalf((g(x[i+1], (j-1)*k)*alfa-e(x[i+1], (j-1)*k)*beta)*u(j, i)
+(1-2*alfa*g(x[i+1], (j-1)*k)-(k1*df(x[i+1], j*k, u[j, i+1])))*u[j, i+1]
+(g(x[i+1], (j-1)*k)*alfa+e(x[i+1], (j-1)*k)*beta)*u(j, i+2)+k1*(f(x[i
+1], j*k, u[j, i+1])+f(x[i+1], (j-1)*k, u[j, i+1])));
        end do;
        # Dosezení okrajových podmínek do první a poslední rovnice
        p := 1/(alfa1*2*h-3*beta1(j*k));
        r := beta1(j*k)*p;
        pom := d1[1]*r;
        d2[1] := d2[1]-4*pom;
        d3[1] := d3[1]+pom;
        F[1] := F[1]-2*h*gama1(j*k)*d1[1]*p;
        q := 1/(alfa2*2*h+3*beta2(j*k));
        s := beta2(j*k)*q;
        pom := d3[n-1]*s;
        d2[n-1] := d2[n-1]+4*pom;
        d1[n-1] := d1[n-1]-pom;
        F[n-1] := F[n-1]-2*h*gama2(j*k)*d3[n-1]*q;
        # Výpočet profilu u - j+1 vrstva, tj. pro t=j*k
        up := TriDiagonalSolve(n-1, d1, d2, d3, F);
        for i from 2 to n do
            u[j+1, i] := up[i-1];
        end do;
```

```
        # výpo¦et okrajových podmínek - j+1 vrstva, tj. pro t=j*k
        u[j+1,1] := 2*h*gama1(j*k)*p-r*(4*u[j+1,2]-u[j+1,3]);
        u[j+1,n1] := 2*h*gama2(j*k)*q+s*(4*u[j+1,n]-u[j+1,n-1]);
    end do;
    RETURN(eval(u));
end proc:
```

PŚíklad 1:

```
phi := x->sin(evalf(Pi·x)):
alfa1 := 1:
 beta1 := t->0:
alfa2 := 1:
 beta2 := t->0:
gama1 := t->0:
 gama2 := t->0:
g := (x,t)->1:
 e := (x,t)->0:
f := (x,t,y)->0:
 n := 10:
m := 80:
 k := 0.005:
h := 1.0/n;
 T := k·m;
```

$$0.1000000000$$

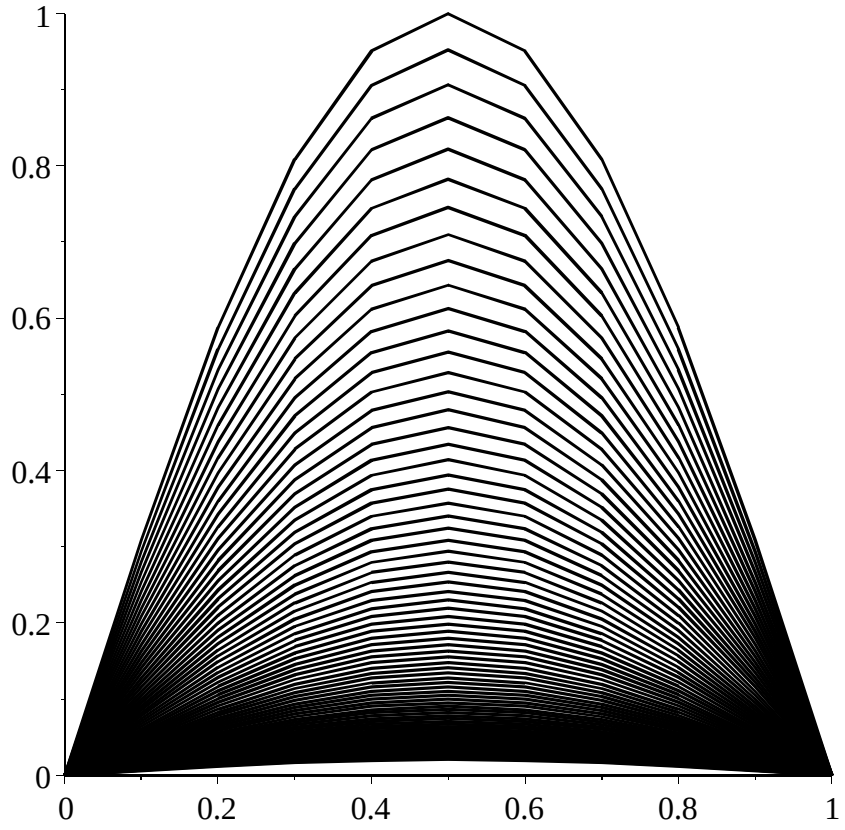
$$0.400 \qquad \textbf{(1.1)}$$

```
vys := PDEParabCN(n, m, k, 0.0, 1.0, g, e, f, alfa1, beta1, alfa2, beta2, gama1, gama2, phi);
```

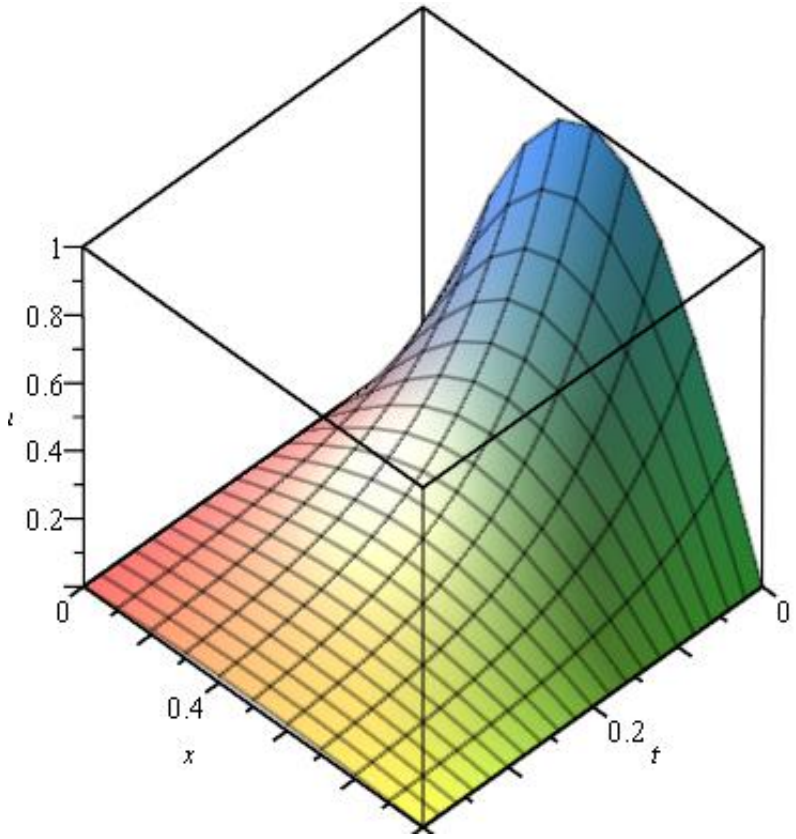
$$\begin{bmatrix} 81 \text{ x } 11 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix} \qquad \textbf{(1.2)}$$

```
data := [seq([seq([0+(i-1)·h, vys[j,i]], i=1..n+1)], j=1..m+1)]:
with(plots):
```

```
display(seq(listplot(data[i]), i=1..m));
```




```
a := subs(1 .. m + 1 = 0 ..T, 1 .. n + 1 = 0 .. 1, matrixplot(vys[1 .. m + 1, 1 .. n + 1], labels = [t,
     x, u])) :
display(a, view = [0 .. T, 0 .. 1, 0 .. 1]);
```



PŚiklad 2

phi $:= x \to 1 - x \cdot x$:
alfa1 $:= 1$:
beta1 $:= t \to 0$:
alfa2 $:= 1$:
beta2 $:= t \to 0$:
gama1 $:= t \to 1$:
gama2 $:= t \to 0$:
$g := (x, t) \to 1$:
$e := (x, t) \to 1$:
$f := (x, t, y) \to -\exp(-y)$:
$n := 10$:
$m := 50$:
$k := 0.005$:
$h := \frac{1.0}{n}$;
$T := k \cdot m$;

0.1000000000

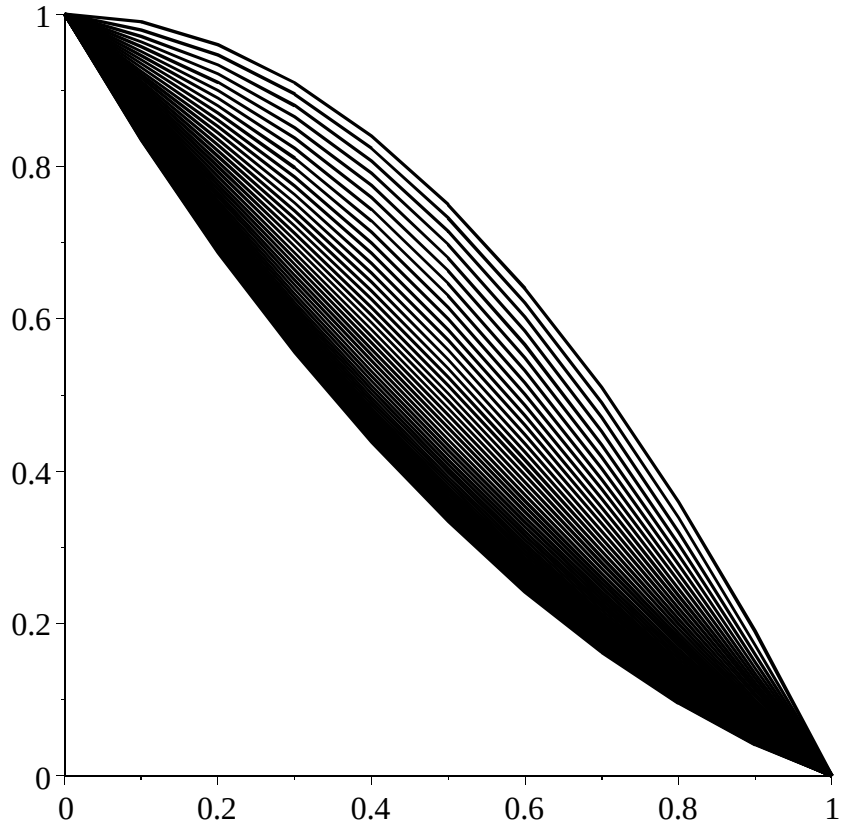
0.250 **(2.1)**

$vys := PDEParabCN(n, m, k, 0.0, 1.0, g, e, f, alfa1, beta1, alfa2, beta2, gama1, gama2, \mathrm{phi});$

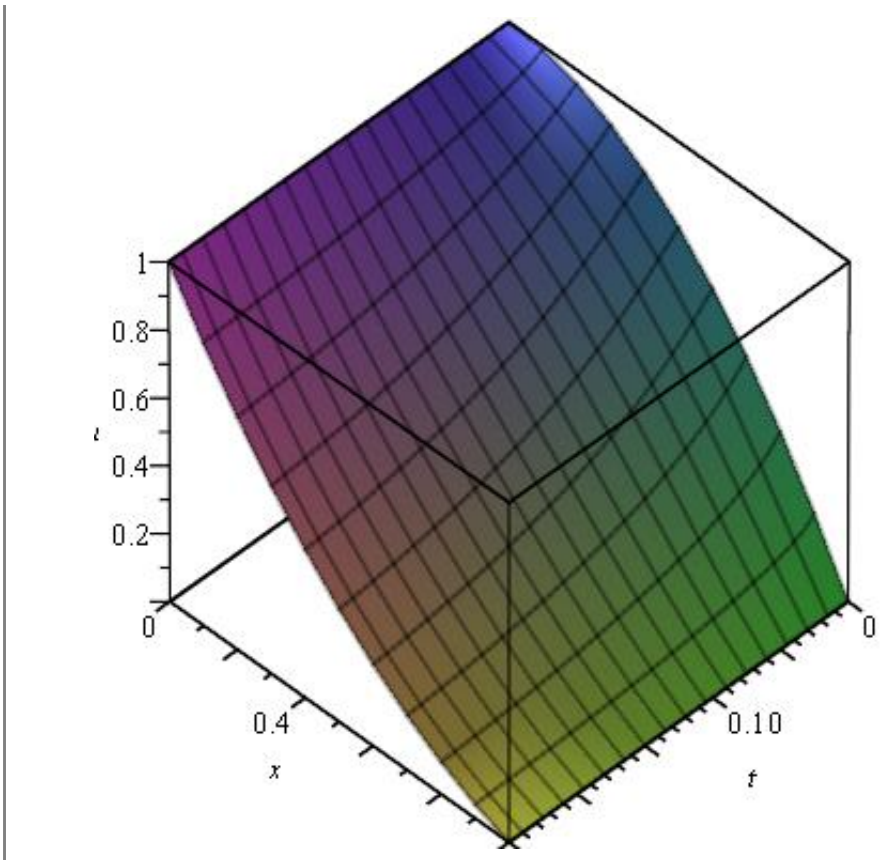
$$\begin{bmatrix} 51 \text{ x } 11 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{bmatrix} \quad \textbf{(2.2)}$$

$data := [seq([seq([0 + (i - 1) \cdot h, vys[j, i]], i = 1 ..n + 1)], j = 1 .. m + 1)]:$
$with(plots):$

$display(seq(listplot(data[\mathrm{i}]), i = 1 ..m));$



$a := subs(1 .. \mathrm{m} + 1 = 0 ..\mathrm{T}, 1 .. \mathrm{n} + 1 = 0 .. 1, matrixplot(vys[1 .. \mathrm{m} + 1, 1 .. \mathrm{n} + 1], labels = [t, x, u])):$
$display(a, view = [0 .. \mathrm{T}, 0 .. 1, 0 .. 1]);$



ř eġení parciální diferenciální rovnice metodou Crank-Nicolsonové s centrální náhradou okrajových podmínek

```
PDEParabCNc := proc (n, m, k, a, b, g, e, f, alfa1, beta1, alfa2, beta2, gama1, gama2, phi)
    local h, n1, m1, u, i, j, alfa, beta, x, F, d1, d2, d3, p, q, r, s, pom1, pom2, up, df, k1;
    with(linalg);
    h := (b-a)/n;
    n1 := n+1;
    m1 := m+1;
    k1 := k/2;
    alfa := evalf(k/(2*h*h));
    beta := evalf(k/(4*h));
    df := D[3](f);
    x := Vector(n1, i -> a+(i-1)*h);
    u := Matrix((m1, n1), 0);
    d1 := Vector(n1, 0);
    d2 := Vector(n1, 0);
    d3 := Vector(n1, 0);
    F := Vector(n1, 0);
    #profil u - 0 vrstva
    for i from 1 to n1 do
        u[1, i] := evalf(phi(x[i]));
```

```
    end do;
    pom1 := phi(a-h);
    pom2 := phi(b+h);
    #Výpočet diagonál v matici A a pravé strany, u(j+1)=F(j)+A*u(j)
    for j from 1 to m do
        for i from 1 to n1 do
            d1[i] := -evalf(g(x[i], j*k)*alfa-e(x[i], j*k)*beta);
            d2[i] := evalf(1+2*alfa*g(x[i], j*k)-k1*df(x[i], j*k, u[j, i]));
            d3[i] := -evalf(g(x[i], j*k)*alfa+e(x[i], j*k)*beta);
        end do;
        for i from 2 to n do
            F[i] := evalf((g(x[i], (j-1)*k)*alfa-e(x[i], (j-1)*k)*beta)*u(j, i-1)+(1
    -2*alfa*g(x[i], (j-1)*k)-(k1*df(x[i], j*k, u[j, i])))*u[j, i]+(g(x[i], (j-1)*k)
    *alfa+e(x[i], (j-1)*k)*beta)*u(j, i+1)+k1*(f(x[i], j*k, u[j, i])+f(x[i], (j-1)
    *k, u[j, i])));
        end do;
        F[1] := evalf((g(x[1], (j-1)*k)*alfa-e(x[1], (j-1)*k)*beta)*pom1+(1-2
    *alfa*g(x[1], (j-1)*k)-k1*df(x[1], j*k, u[j, 1]))*u[j, 1]+(g(x[1], (j-1)*k)*alfa
    +e(x[1], (j-1)*k)*beta)*u[j, 2]+k1*(f(x[1], j*k, u[j, 1])+f(x[1], (j-1)*k, u[j,
    1])));
        F[n1] := evalf((g(x[n1], (j-1)*k)*alfa-e(x[n1], (j-1)*k)*beta)*u[j, n]+(1-2
    *alfa*g(x[n1], (j-1)*k)-k1*df(x[n1], j*k, u[j, n1]))*u[j, n1]+(g(x[n1], (j-1)*k)
    *alfa+e(x[n1], (j-1)*k)*beta)*pom2+k1*(f(x[n1], j*k, u[j, n1])+f(x[n1], (j-1)
    *k, u[j, n1])));
        # Dosezení okrajových podmínek do první a poslední rovnice
        if beta1(t)=0 then d2[1] := alfa1; d3[1] := 0; F[1] := gama1(j*k) else
        p := 2*h/beta1(j*k);
        d2[1] := d2[1]+alfa1*d1[1]*p;
        d3[1] := d3[1]+d1[1];
        F[1] := F[1]+gama1(j*k)*d1[1]*p;
        end if;
        if beta2(t)=0 then  d2[n1] := alfa2; d1[n1] := 0; F[n1] := gama2(j*k) else
        q := 2*h/beta2(j*k);
        d2[n1] := d2[n1]-alfa2*d3[n1]*q;
        d1[n1] := d1[n1]+d3[n1];
        F[n1] := F[n1]-gama2(j*k)*d3[n1]*q;
        end if;
        # Výpočet profilu u - j+1 vrstva, tj. pro t=j*k
        up := TriDiagonalSolve(n1, d1, d2, d3, F);
        for i from 1 to n1 do
            u[j+1, i] := up[i];
        end do;
        # výpočet pomocných okrajových hodnot - j+1 vrstva, tj. pro t=j*k
        if beta1(t)=0 then pom1 := gama1(j*k)/alfa1 else
        pom1 := (-gama1(j*k)+alfa1*u[j+1, 1])*p+u[j+1, 2]
        end if;
        if beta2(t)=0 then pom2 := gama2(j*k)/alfa2 else
        pom2 := (gama2(j*k)-alfa2*u[j+1, n1])*p+u[j+1, n]
        end if;
    end do;
    RETURN(eval(u));
end proc:
```

PŚíklad 1:

```
phi := x→sin(evalf(Pi·x)):
alfa1 := 1:
 beta1 := t→0:
alfa2 := 1:
 beta2 := t→0:
gama1 := t→0:
 gama2 := t→0:
g := (x, t)→1:
 e := (x, t)→0:
f := (x, t, y)→0:
 n := 10:
m := 80:
 k := 0.005:
h := 1.0/n;
 T := k·m;
```

$$0.1000000000$$

$$0.400 \qquad \textbf{(3.1)}$$

```
vys := PDEParabCNc(n, m, k, 0.0, 1.0, g, e, f, alfa1, beta1, alfa2, beta2, gama1, gama2, phi);
```

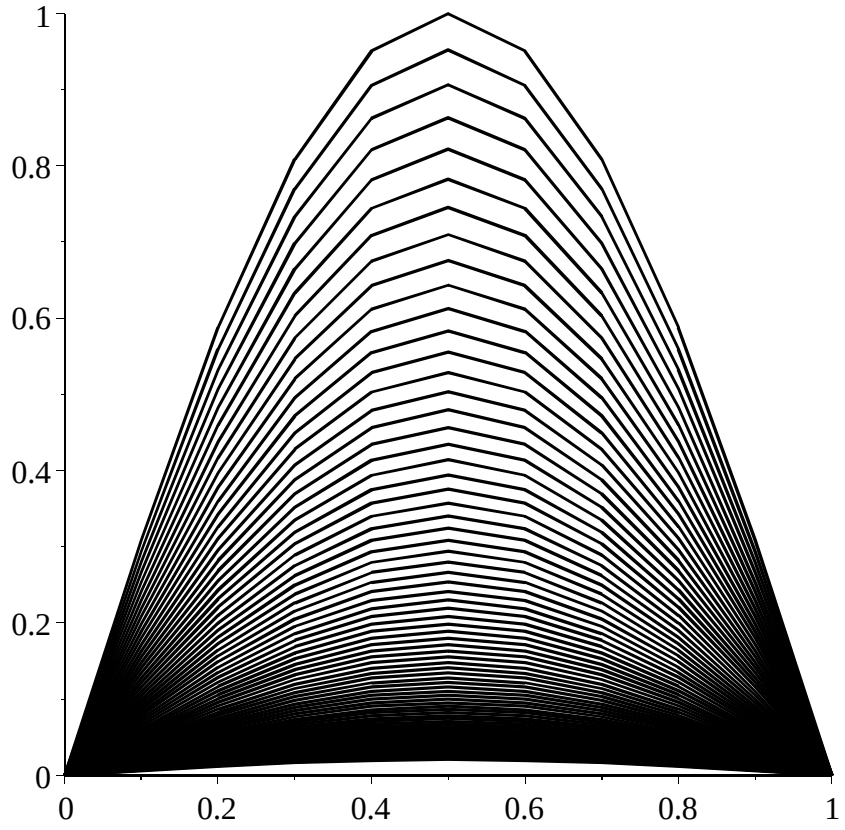
$$\begin{bmatrix} \textit{81 x 11 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix} \qquad \textbf{(3.2)}$$

(3.3)

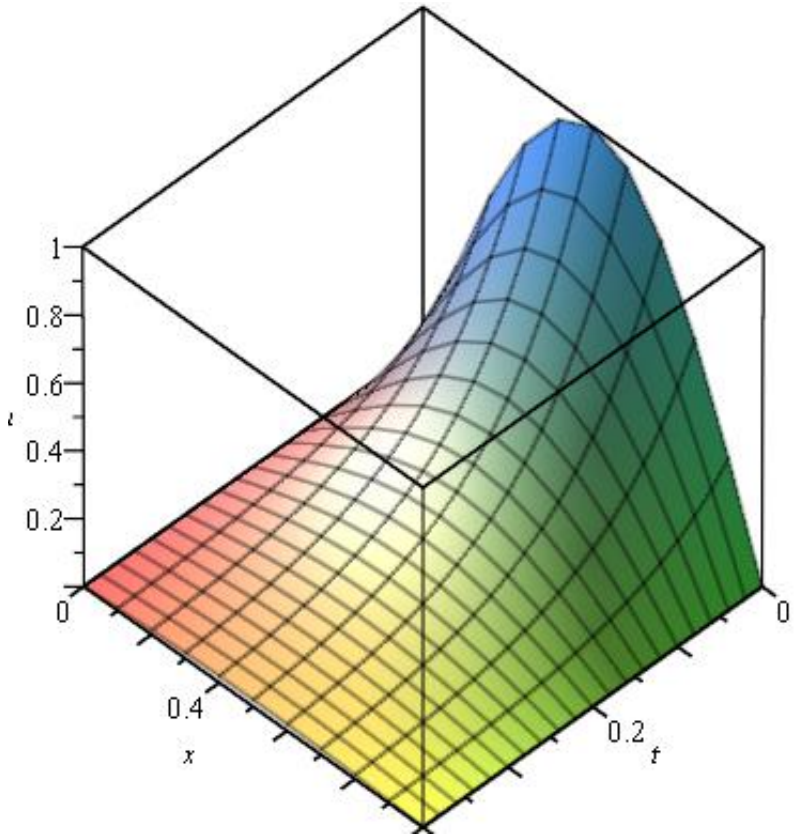
```
data := [seq([seq([0 + (i − 1)·h, vys[j, i]], i = 1 ..n + 1)], j = 1 .. m + 1)]:
with(plots):



display(seq(listplot(data[i]), i = 1 ..m));
```




```
a := subs(1 .. m + 1 = 0 ..T, 1 .. n + 1 = 0 .. 1, matrixplot(vys[1 .. m + 1, 1 .. n + 1], labels = [t,
     x, u])) :
display(a, view = [0 .. T, 0 .. 1, 0 .. 1]);
```



PŚíklad 2

phi := $x \to 1 - x \cdot x$:
alfa1 := 1 :
beta1 := $t \to 0$:
alfa2 := 1 :
beta2 := $t \to 0$:
gama1 := $t \to 1$:
gama2 := $t \to 0$:
$g := (x, t) \to 1$:
$e := (x, t) \to 1$:
$f := (x, t, y) \to -\exp(-y)$:
$n := 10$:
$m := 50$:
$k := 0.005$:
$h := \frac{1.0}{n}$;
$T := k \cdot m$;

0.1000000000

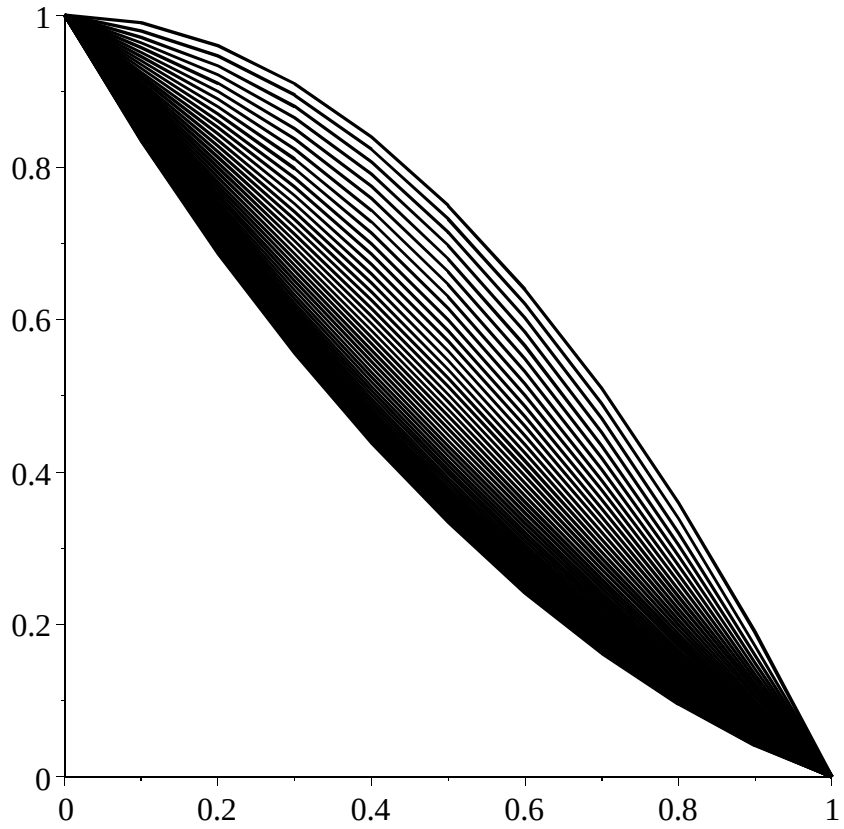
0.250 **(4.1)**

vys := *PDEParabCNc*(*n*, *m*, *k*, 0.0, 1.0, *g*, *e*, *f*, *alfa1*, *beta1*, *alfa2*, *beta2*, *gama1*, *gama2*, phi);

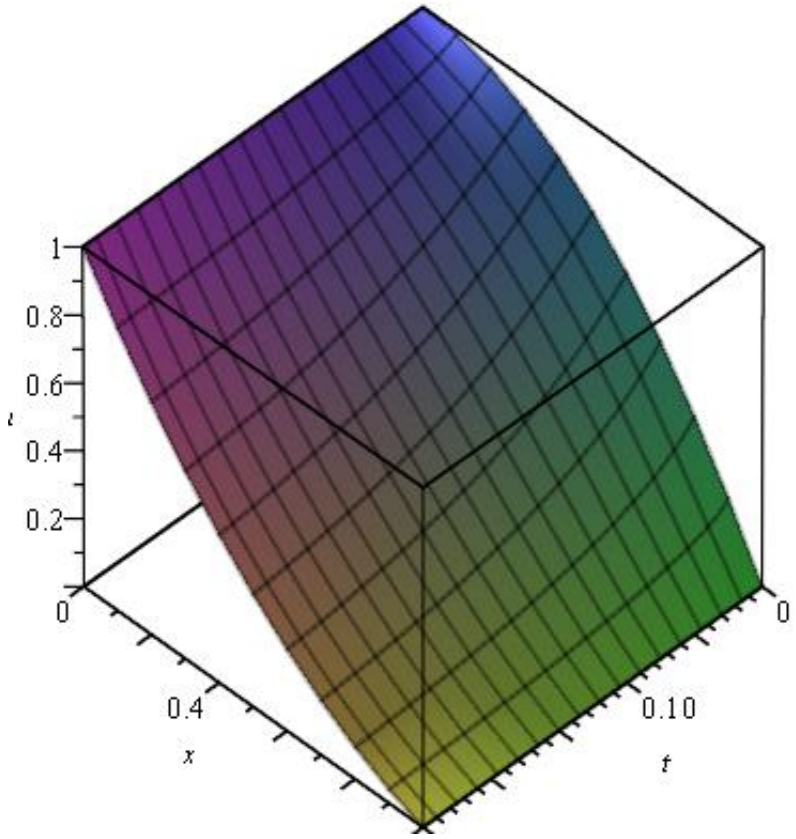
$$\begin{bmatrix} \textit{51 x 11 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix} \quad \textbf{(4.2)}$$

$data := [seq([seq([0 + (i-1)\cdot h, vys[j, i]], i = 1..n+1)], j = 1..m+1)]:$
$with(plots):$

$display(seq(listplot(data[i]), i = 1..m));$



$a := subs(1..m+1 = 0..T, 1..n+1 = 0..1, matrixplot(vys[1..m+1, 1..n+1], labels = [t, x, u]))):$
$display(a, view = [0..T, 0..1, 0..1]);$


1
0.8
0.6
u
0.4
0.2
0
0.4
x
0.10
t
0