CTU
CZECH TECHNICAL UNIVERSITY IN PRAGUE

Stlačitelné proudění, segregovaná a sdružená metoda

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Compressible flows & OF

Segregated OF solvers for compressible flows:

SIMPLE based solvers: *rhoSimpleFoam*, *rhoPimpleFoam*

- different setup for subsonic and transonic/supersonic flows:
  - compressibility effects in the continuity equation
  - different relaxation factors (problem dependent?)
- very sensitive to initial condition
- low efficiency for transonic/supersonic flows

PISO based solvers: *sonicFoam*, hybrid AUSM+ scheme of [Xisto]

- limited Courant number → limited efficiency for steady state cases

Coupled (density based) OF solvers:

- *rhoCentralFoam*: explicit central-upwind scheme by Tadmor
- *dbns* solvers (Foam-extend): explicit Riemann solvers based scheme
- *aeroFoam*: (?) similar to *dbns*

✖ **All coupled OF schemes use explicit time stepping → strong limit on *Δt*!**

Segregovaný řešič, algoritmus SIMPLE

Rovnice pro tlak z rovnice kontinuity

$$U = \hat{U} - \frac{1}{a}\nabla p$$

$$\rho = \psi p$$

$$\psi = \frac{\rho}{p} = \frac{1}{rT}$$

$$\nabla \cdot (\rho U) = \nabla \cdot (\psi \hat{U} \mathbf{p}) - \nabla \cdot (\frac{\rho}{a} \nabla \mathbf{p}) = 0$$

$$\psi = const. \Rightarrow T = const.$$

Jak odpovídá předpoklad *T=const.* rovnici energie?

$$H = c_p T + \frac{1}{2}||U||^2 = const.$$

$$T' \approx -\frac{1}{c_p} U \cdot U'$$

$$\frac{T'}{T} \approx -\frac{U^2}{c_p T}\frac{U'}{U} = (1-\gamma)M^2 \frac{U'}{U}$$

Segregovaný řešič

Eulerovy rovnice v 1D

$$\rho_t + (\rho u)_x = 0$$
$$(\rho u)_t + (\rho u^2 + p)_x = 0$$
$$(\rho E)_t + (\rho u H)_x = 0$$

$$p = \rho r T$$
$$E = c_v T + u^2/2$$
$$H = c_p T + u^2/2$$

Linearizace (stacionární případ)

$$\rho'_t + \rho'_x \bar{u} + \bar{\rho} u'_x = 0$$
$$u'_t + \bar{u} u'_x + \frac{1}{\bar{\rho}} p'_x = 0$$
$$p'_t + \bar{u} p'_x + \kappa \bar{p} u'_x = 0$$

$$\frac{\rho'_x}{\bar{\rho}} + \frac{u'_x}{\bar{u}} = 0$$
$$\frac{u'_x}{\bar{u}} + \frac{\bar{p}}{\bar{\rho}\bar{u}^2} \frac{p'_x}{\bar{p}} = 0$$
$$\frac{u'_x}{\bar{u}} + \kappa \frac{p'_x}{\bar{p}} = 0$$

Algoritmus SIMPLE

$$\frac{u'}{\bar{u}} \approx -\frac{1}{a} \frac{\kappa}{M^2} \frac{p'_x}{\bar{p}},$$

$$\bar{u} \frac{p'_x}{\bar{p}} - \left(\frac{\kappa}{a M^2} \frac{p'_x}{\bar{p}} \right)_x = 0,$$

=> relaxace závislá na *M*

LU-SGS Solver for OF

Previous works:
- [Gill et al, 2013] – LU-SGS for steady turbulent flows (at OFW8)
- [Heyns et al, 2014] – GMRES/LU-SGS solver
- [Shen et al, 2016] – detailed description of the implementation, dual time stepping for unsteady cases

Current work:
- matrix-free LU-SGS
- based on *dbns* library with following improvements:
  - ✓ run-time selection of the Riemann solver (AUSM+up, HLLC, ...)
  - ✓ support for dynamic meshes (arbitrary Lagrangian-Eulerian method, multiple reference frames)
- unified solver for steady and transient case

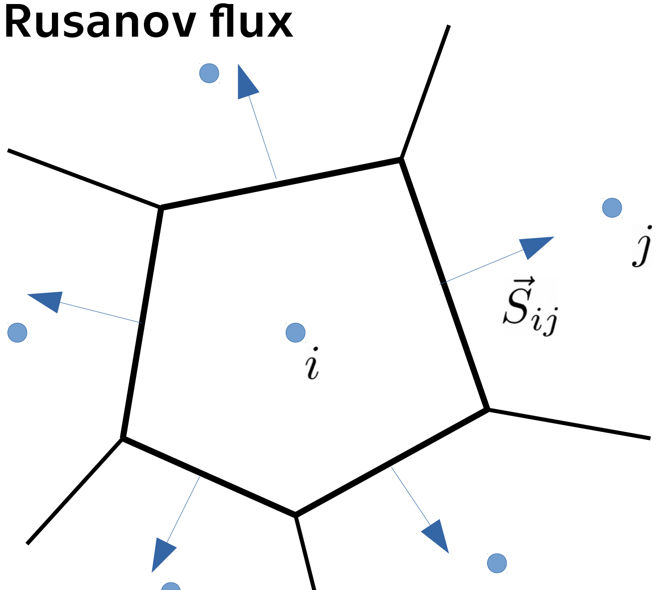
LU-SGS Solver for OF

- Navier-Stokes equations for compressible flows
- AUSM and HLLC fluxes
- central approximation of viscous fluxes
- low-order Jacobian based on Rusanov flux

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_j)}{\partial x_j} = 0$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} + \frac{\partial p}{\partial x_i} = \frac{\partial \tau_{ij}^{eff}}{\partial x_j}$$

$$\frac{\partial(\rho E)}{\partial t} + \frac{\partial\left[(\rho E + p)u_j\right]}{\partial x_j} = \frac{\partial(u_i \tau_{ij}^{eff} - q_j^{eff})}{x_j}$$



$$W = [\rho, \rho\vec{u}, \rho E]$$

$$|\Omega_i| \frac{dW_i}{dt} = -R(W)_i = -\sum_j (F_{ij} - F_{ij}^v)$$

$$|\Omega_i| \frac{W_i^{n+1} - W_i^n}{\Delta t} \approx -R(W^n)_i - \frac{\partial R^{lo}}{\partial W_j} (W_j^{n+1} - W_j^n)$$

$$\sum_j \left[\frac{|\Omega_i|}{\Delta t} \delta_{ij} + \frac{\partial R_i^{lo}}{\partial W_j} \right] \Delta W_j = -R(W^n)_i.$$

LU-SGS Solver for OF

Lower-Upper Symmetric Gauss Seidel method for *Ax=b*:

$$A = L + D + U \approx (L+D)D^{-1}(D+U)$$

$$D\Delta x^* = b - Ax^n - L\Delta x^*,$$
$$D\Delta x = D\Delta x^* - U\Delta x,$$
$$x^{n+1} = x^n + \Delta x.$$

Matrix-free LU-SGS for compressible flows [Blazek]:

$$D_i \Delta W_i^* = -\tilde{R}(W^n)_i - \frac{1}{2}\sum_{j<i}\left[\Delta \mathbb{F}_j^* \cdot \vec{S}_{ij} + \lambda_{ij}\Delta W_j^*\right],$$ **for i=0,…,#cells,**

$$D_i \Delta W_i = D_i \Delta W_i^* - \frac{1}{2}\sum_{j>i}\left[\Delta \mathbb{F}_j \cdot \vec{S}_{ij} + \lambda_{ij}\Delta W_j\right],$$ **for i=#cells,…,0.**

where:

$$\Delta \mathbb{F}_j^* = \mathbb{F}(\vec{W}_j^*) - \mathbb{F}(\vec{W}_j^n),$$
$$\Delta \mathbb{F}_j = \mathbb{F}(\vec{W}_j^{n+1}) - \mathbb{F}(\vec{W}_j^n),$$

$$\mathbb{F}(W) = \begin{bmatrix} \rho\vec{u} \\ \rho\vec{u}\otimes\vec{u} + p\mathbb{I} \\ (\rho E + p)\vec{u} \end{bmatrix}, \quad D_i = \left(\frac{|\Omega_i|}{\Delta\tau_i} + \frac{1}{2}\sum_{j\in N_i}\lambda_{ij}\right) I,$$

$$\lambda_{ij} = \omega\left[|\vec{U}_{ij}\cdot\vec{S}_{ij}| + |\vec{S}_{ij}|a_{ij} + \frac{|\vec{S}_{ij}|}{||\vec{x}_i - \vec{x}_j||}\max\left(\frac{4}{3\rho_{ij}}, \frac{\gamma}{\rho_{ij}}\right)\left(\frac{\mu}{Pr} + \frac{\mu_T}{Pr_T}\right)\right].$$

Boundary conditions

"Standard" OF boundary conditions at subsonic inlet:

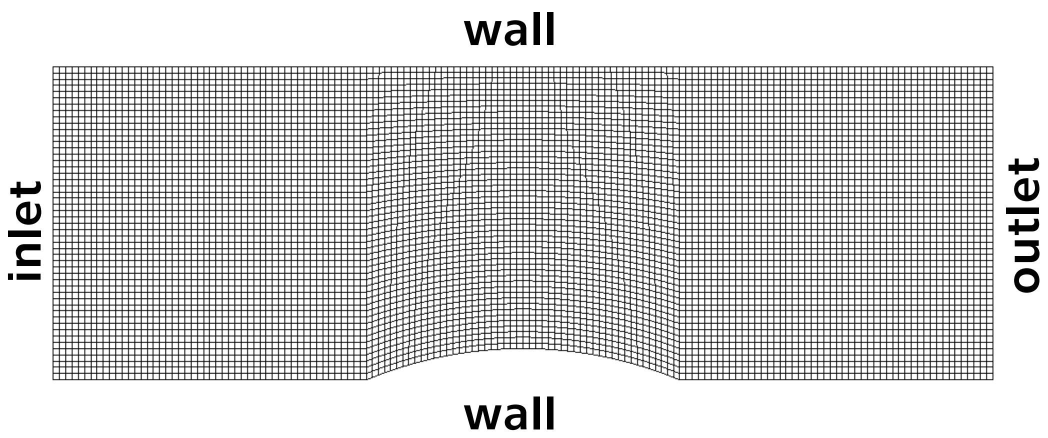
- p_{tot} – implemented as *totalPressure* → $p_f = f(U_f, p_{tot})$
- T_{tot} – implemented as *totalTemperature* → $T_f = f(U_f, T_{tot})$
- α – implemented as *pressureDirectedVelocity* → $U_f = f(\varphi_f, \alpha)$

OK for segregated solvers where φ is updated from the flux of *pEqn*, but not compatible with coupled solvers where $\varphi = f(U)$!

New boundary condition for U at subsonic inlet:

1) $U_f = f(p_{int}, p_{tot}, T_{tot}, \alpha)$,
2) $p_f = f(U_f, p_{tot})$, (*totalPressure*)
3) $T_f = f(U_f, T_{tot})$, (*totalTemperature*)
4) $\varphi_f = f(U_f)$.

2D flow over a bump



Channel with circular bump
length: 3m
height: 1m
bump height:
0.1m for subsonic inlets
0.04m for supersonic inlets

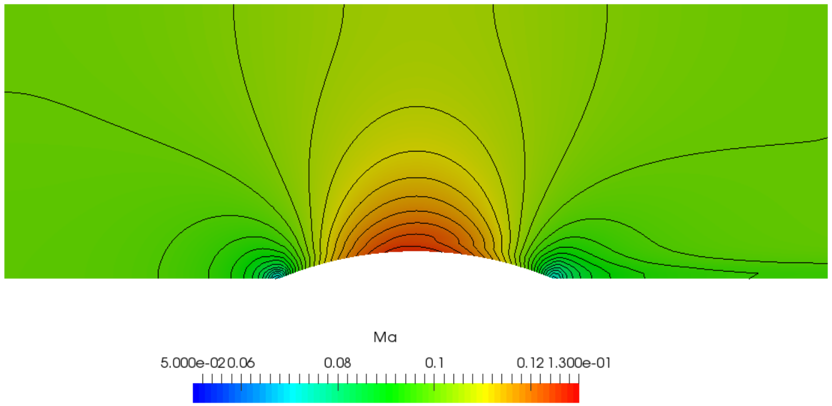
Test cases:
subsonic: M_{in}=0.1
transonic: M_{in}=0.675
supersonic: M_{in}=1.65

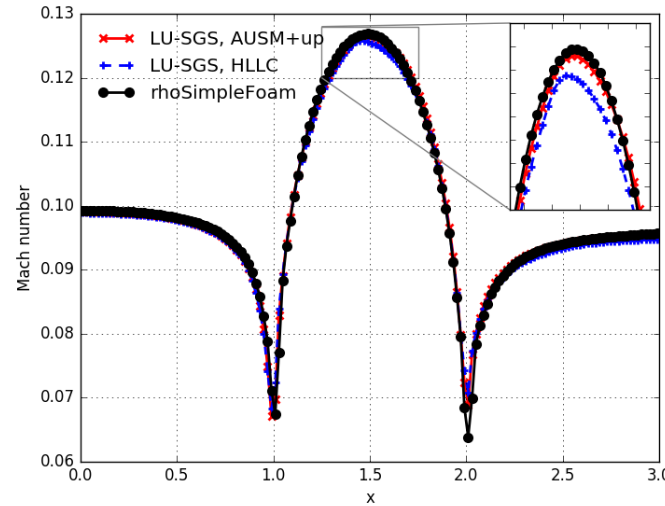
Mesh:
structured
coarse with 150x50 cells
fine with 450x150 cells

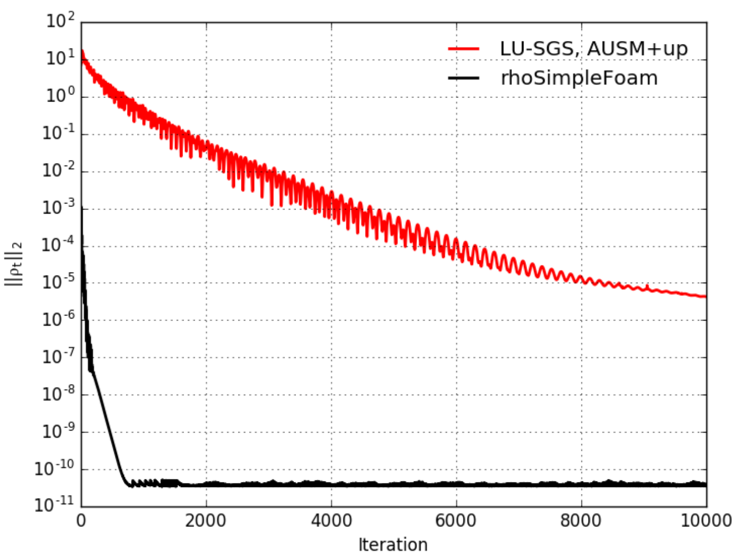
Flow field:
2D
compressible ideal gas
inviscid

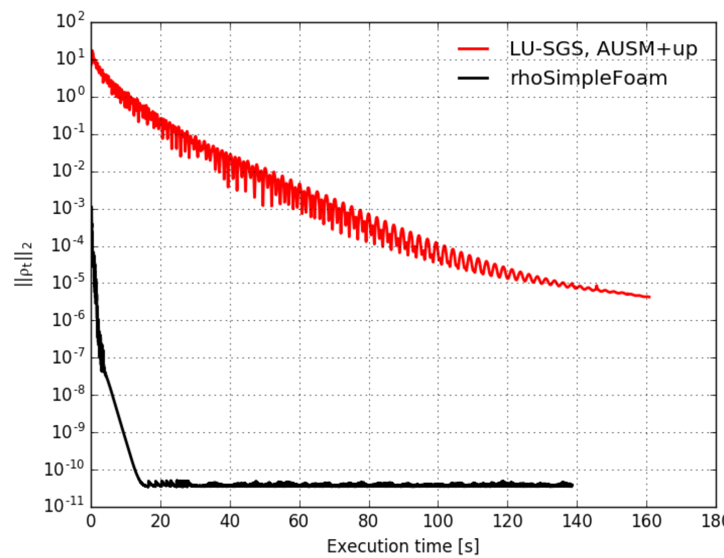
Solution obtained with
pressure correction method (rhoSimpleFOAM)
LU-SGS with AUSM+up or HLLC scheme using Barth or Venkatakrishnan limiter

Flow over a bump, M_{in}=0.1

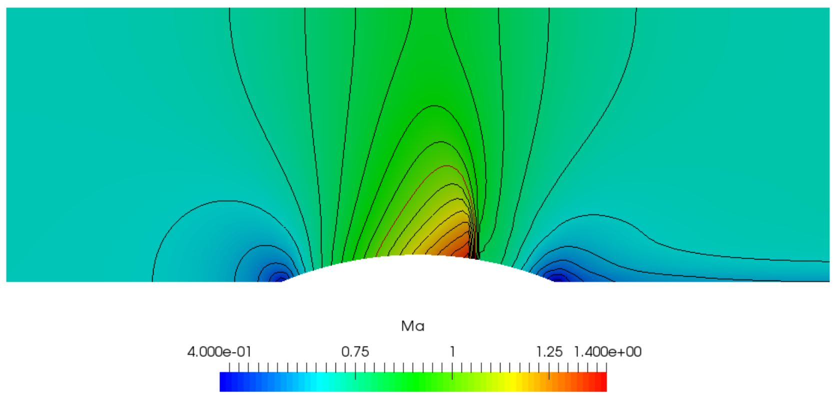


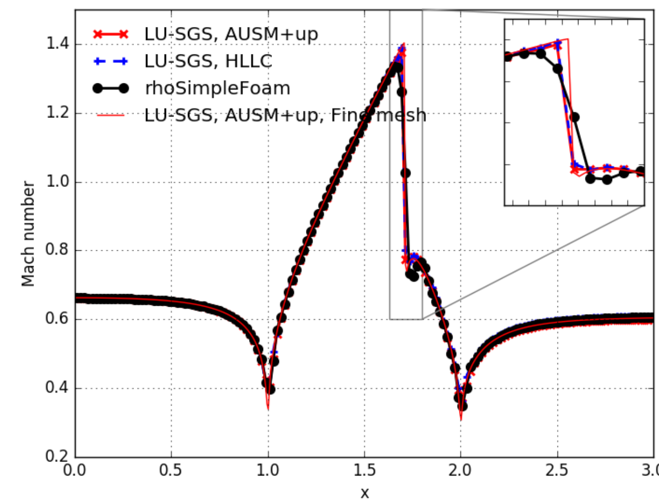


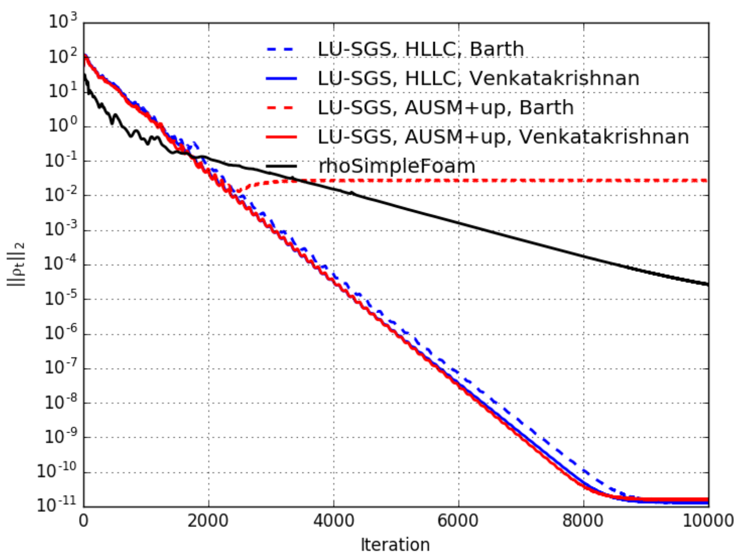


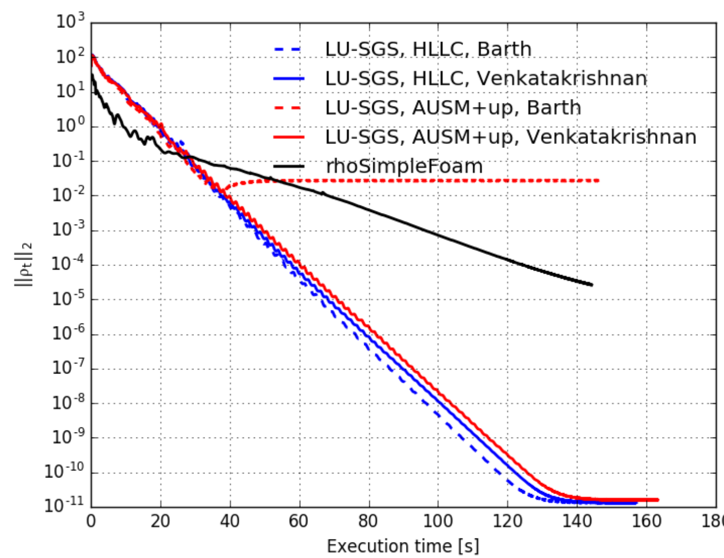


Flow o. a bump, M_{in}=0.675

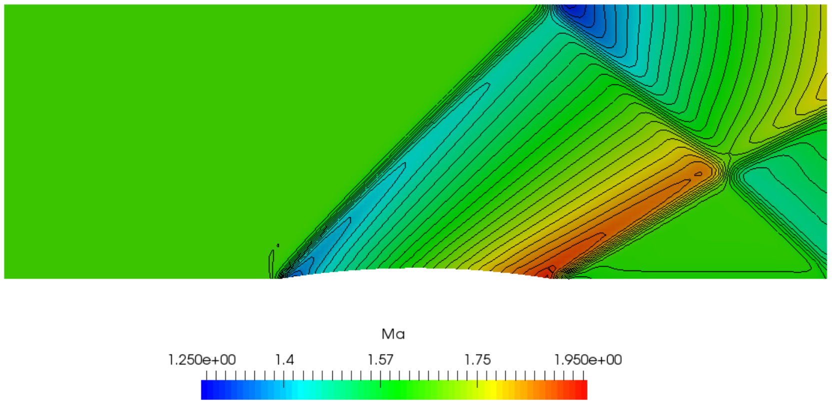


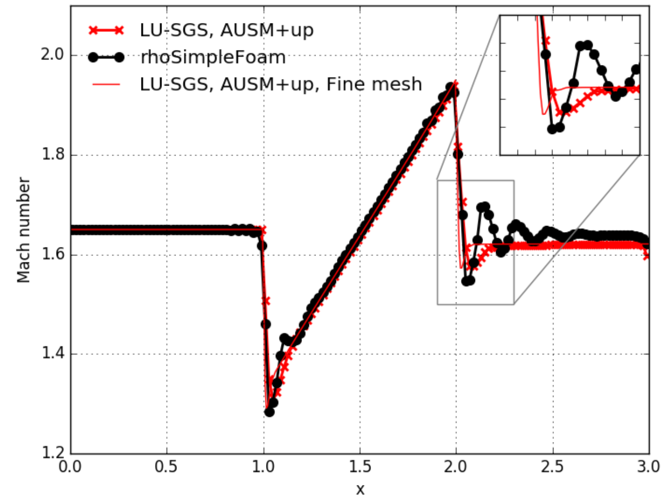


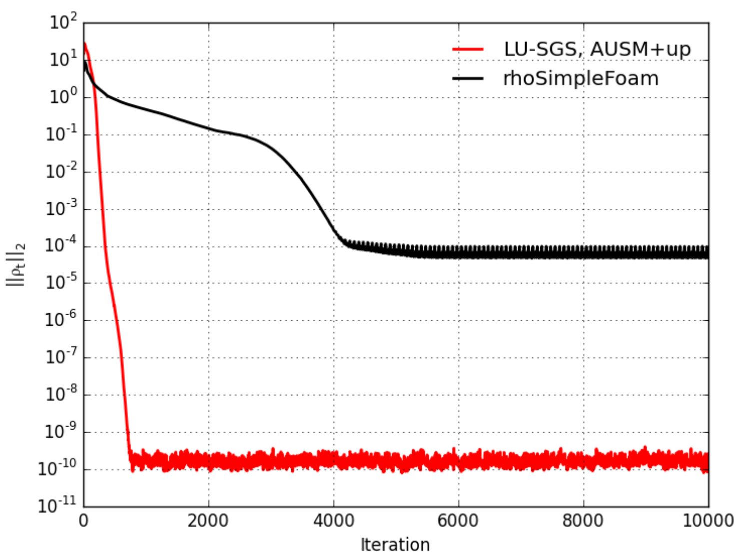


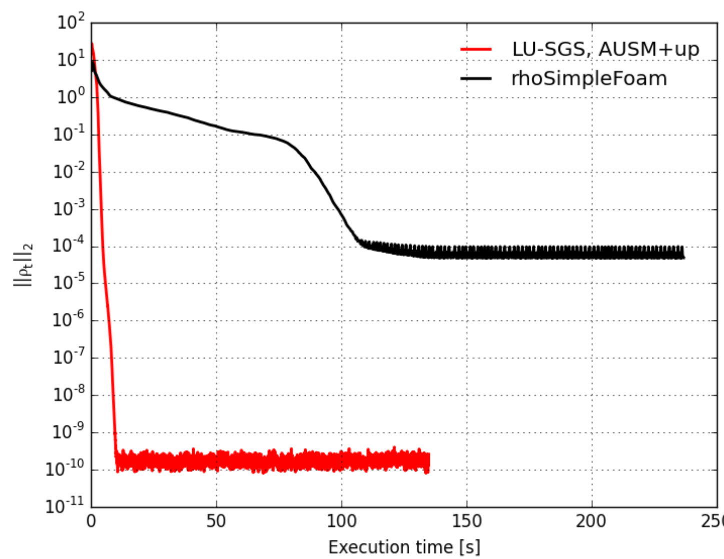


Flow o. a bump, M_{in}=1.65





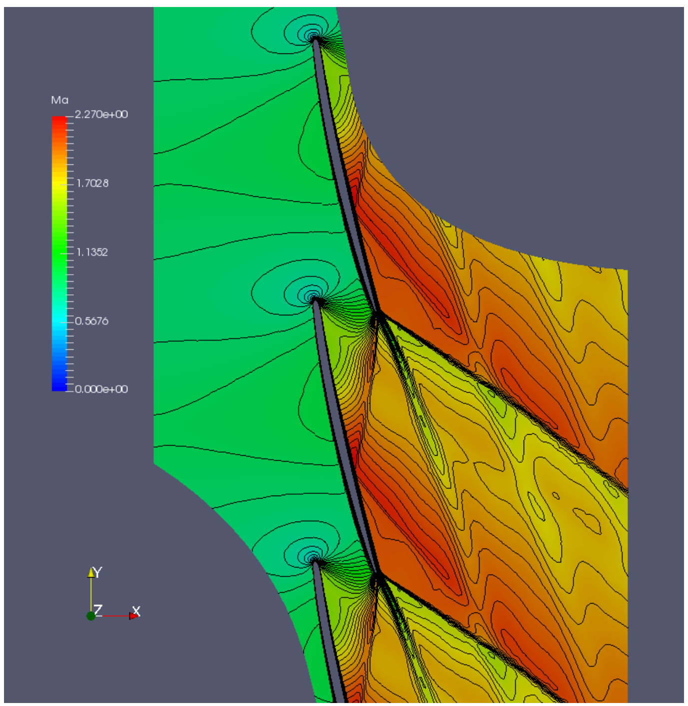




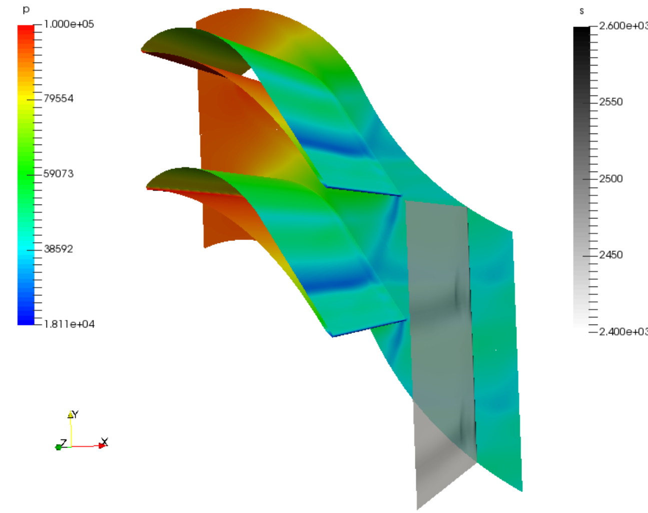
Applications

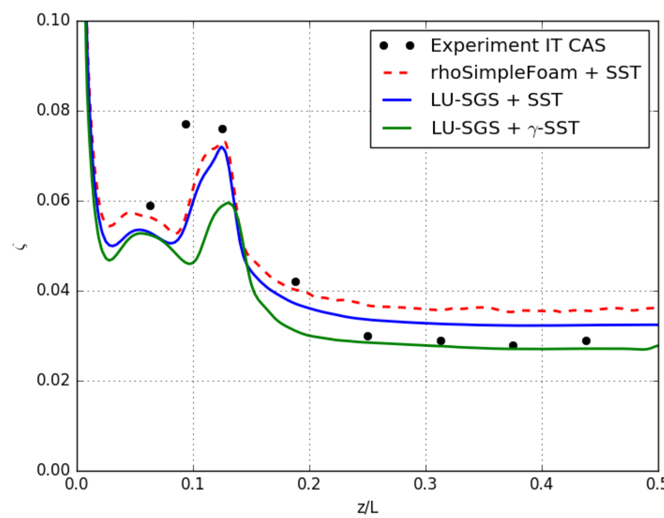
2D flows through tip section of a turbine cascade

- $M_{out} \sim 2$
- $Re \sim 10^6$



3D flows through turbine cascade

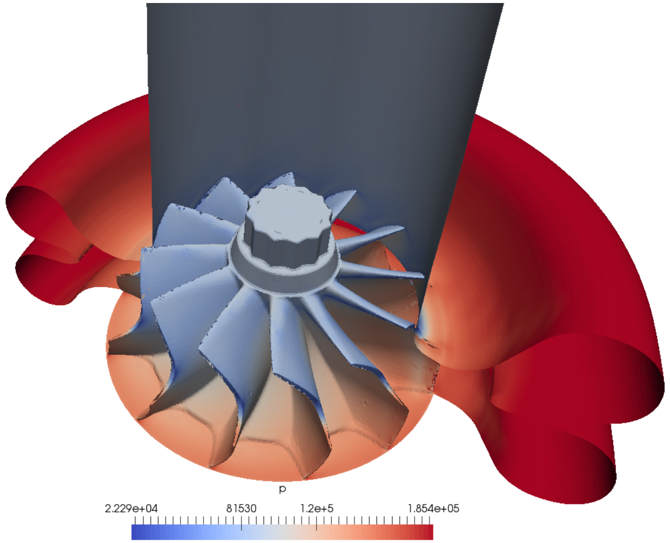


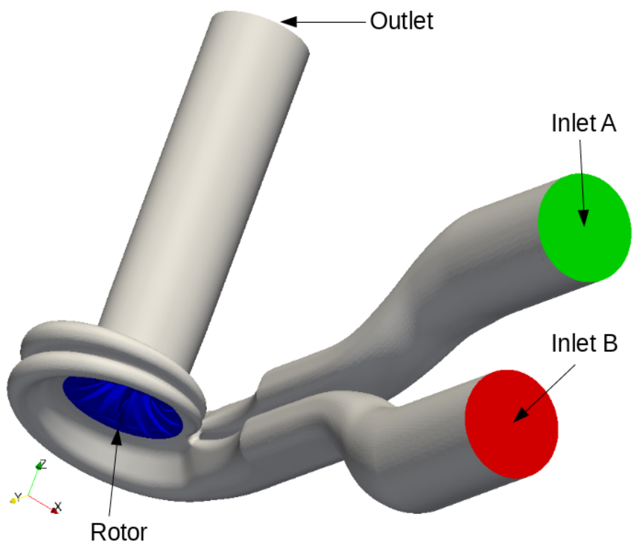


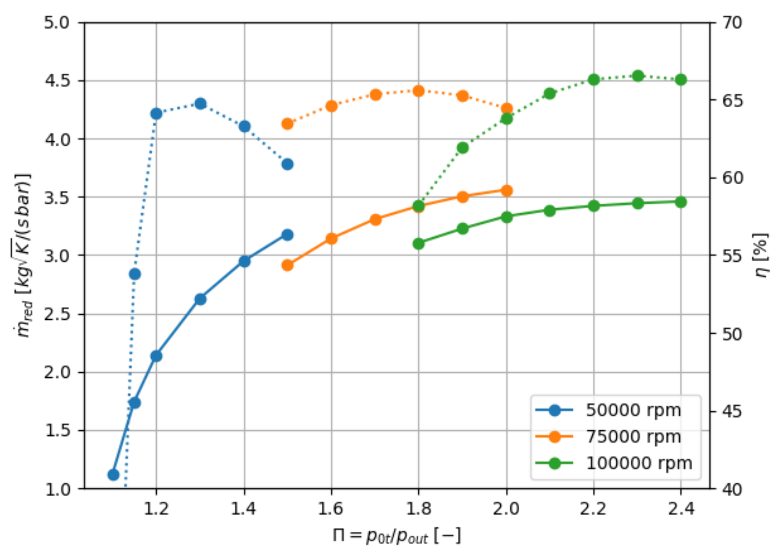
Applications

Analysis of twin-scroll turbine

- high rotation speed (50-100 krpm)
- expansion ratio Π=1.2-2.2
- important compressibility
- steady (MRF) / transient simulation
- 2 mil. cells, snappyHexMesh







Conclusions

The matrix-free LU-SGS method has been implemented for steady and unsteady turbulent flows (using MRF or dynamic mesh).

The LU-SGS method is:

- very simple
- efficient for compressible flows (Ma>0.3)
- provides sharp resolution of shock waves
- has very low memory footprint

The LU-SGS solver is compatible with the rest of OpenFOAM framework (turbulence models, parallel processing, …)

Future work:

- preconditioning for low Mach number flows

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