

$$w = e^{-i\frac{2\pi}{N}}$$

$$\sum_k = \sum_{k=0}^N$$

$$\hat{a}_m = \frac{1}{\sqrt{N}} \sum_k a_k w^{mk},$$

$$\hat{b}_m = \frac{1}{\sqrt{N}} \sum_k b_k w^{mk},$$

$$(a * b)_m = \frac{1}{\sqrt{N}} \sum_k a_k b_{m-k}.$$

$$\widehat{a * b} = \hat{a} \hat{b}$$

$$\widehat{a * b}_m = \hat{a}_m \hat{b}_m$$

$$\begin{aligned} L &= \widehat{a * b}_m \\ &= \frac{1}{\sqrt{N}} \sum_k (a * b)_k w^{mk} \\ &= \frac{1}{\sqrt{N}} \sum_k \left(\frac{1}{\sqrt{N}} \sum_j a_j b_{k-j} \right) w^{mk} \\ &= \frac{1}{\sqrt{N}} \sum_j \frac{1}{\sqrt{N}} \sum_k a_j b_{k-j} w^{mk} \\ &= \frac{1}{\sqrt{N}} \sum_j a_j \frac{1}{\sqrt{N}} \sum_k b_{k-j} w^{mk} \quad |k - j = r| \\ &= \frac{1}{\sqrt{N}} \sum_j a_j \frac{1}{\sqrt{N}} \sum_r b_r w^{m(j+r)} \\ &= \left(\frac{1}{\sqrt{N}} \sum_j a_j w^{mj} \right) \left(\frac{1}{\sqrt{N}} \sum_r b_r w^{mr} \right) \\ &= \hat{a}_m \hat{b}_m \\ &= R. \end{aligned}$$