

$$T = 2\pi$$

$$\sum_n = \sum_{n \in \mathbb{Z}}$$

$$(f * g)(x) = \frac{1}{T} \int_0^T f(y) g(x-y) dy$$

$$f(t) = \sum_n \hat{f}_n e^{int}$$

$$(\hat{f} * \hat{g})_n = \sum_k \hat{f}_k \hat{g}_{n-k}$$

$$\hat{f}_n = \frac{1}{T} \int_0^T f(t) e^{-int} dt$$

$$a) \widehat{f * g} = \hat{f} \cdot \hat{g}$$

$$(\widehat{f * g})_n = \hat{f}_n \cdot \hat{g}_n$$

$$L = (\widehat{f * g})_n = \frac{1}{T} \int_0^T (f * g)(t) e^{-int} dt = \frac{1}{T} \int_0^T \frac{1}{T} \int_0^T f(y) g(t-y) dy e^{-int} dt =$$

$$= \frac{1}{T} \int_0^T \frac{1}{T} \int_0^T f(y) g(t-y) e^{-int} dt dy = \frac{1}{T} \int_0^T f(y) \frac{1}{T} \int_0^T g(t-y) e^{-int} dy dt = \left| t-y=\tau \right|$$

$$= \frac{1}{T} \int_0^T f(y) \frac{1}{T} \int_0^T g(\tau) e^{-im(y+\tau)} d\tau dy = \frac{1}{T} \int_0^T f(y) e^{-imy} \frac{1}{T} \int_0^T g(\tau) e^{-im\tau} d\tau dy =$$

$$= \frac{1}{T} \int_0^T f(y) e^{-imy} dy \cdot \frac{1}{T} \int_0^T g(\tau) e^{-im\tau} d\tau = \hat{f}_n \cdot \hat{g}_n = R$$

$$b) \widehat{f \cdot g} = \hat{f} * \hat{g}$$

$$(\widehat{f \cdot g})_n = \hat{f}_n * \hat{g}_n$$

$$\text{For } k, m, n \in \mathbb{Z} \quad \frac{1}{T} \int_0^T e^{it(k+m-n)} dt = \begin{cases} 1 & \text{for } k+m=n \\ 0 & \text{for } k+m \neq n \end{cases}$$

$$L = (\widehat{f \cdot g})_n = \frac{1}{T} \int_0^T f(t) g(t) e^{-int} dt = \frac{1}{T} \int_0^T \sum_k \hat{f}_k e^{ikt} \sum_m \hat{g}_m e^{imt} e^{-int} dt =$$

$$= \sum_k \sum_m \hat{f}_k \hat{g}_m \frac{1}{T} \int_0^T e^{it(k+m-n)} dt = \sum_k \hat{f}_k \hat{g}_{n-k} = (\hat{f} * \hat{g})_n$$