$$F=\int\sqrt{1+x^2}\,dx=\left|\begin{array}{ccc} x & = & \tan t\\ dx & = & \frac{1}{\cos^2 t}dt\end{array}\right|=\int\sqrt{1+\frac{\sin^2 t}{\cos^2 t}}\,\frac{1}{\cos^2 t}\,dt=$$

$$=\int\frac{1}{\cos^3 t}\,dt=\int\frac{\cos t}{\cos^4 t}\,dt=\left|\begin{array}{ccc} u & = & \sin t\\ du & = & \cos t dt\end{array}\right|=\int\frac{1}{(1-u^2)^2}\,du=\int\frac{1}{(u^2-1)^2}\,du=$$

$$=\frac{1}{4}\int\left(\frac{1}{u+1}-\frac{1}{u-1}+\frac{1}{(u+1)^2}+\frac{1}{(u-1)^2}\right)\,du$$

$$=\frac{1}{4}\log\left|\frac{u+1}{u-1}\right|-\frac{1}{4}\left(\frac{1}{u+1}+\frac{1}{u-1}\right)=\frac{1}{4}\log\left|\frac{u+1}{u-1}\right|+\frac{1}{4}\frac{2u}{1-u^2}=$$

$$=\frac{1}{4}\log\left|\frac{1+\sin t}{1-\sin t}\right|+\frac{1}{2}\frac{\sin t}{\cos^2 t}=$$

$$=\frac{1}{4}\log\left|\frac{(1+\sin t)(1+\sin t)}{(1-\sin t)(1+\sin t)}\right|+\frac{1}{2}\frac{\sin t}{\cos^2 t}=$$

$$=\frac{1}{4}\log\left|\frac{(1+\sin t)^2}{\cos^2 t}\right|+\frac{1}{2}\frac{\sin t}{\cos^2 t}=$$

$$=\frac{1}{2}\log\left|\frac{1+\sin t}{\cos t}\right|+\frac{1}{2}\frac{\sin t}{\cos^2 t}=$$

$$=\frac{1}{2}\log\left|\frac{1}{\cos t}+\tan t\right|+\frac{1}{2}\frac{1}{\cos t}\tan t=$$

$$\left|\frac{1}{\cos t}=\sqrt{1+x^2}\right|$$

$$=\frac{1}{2}\log\left(x+\sqrt{1+x^2}\right)+\frac{1}{2}x\sqrt{1+x^2}=$$

$$=\frac{1}{2}\operatorname{arcsinh}x+\frac{1}{2}x\sqrt{1+x^2}$$

$$F'=\frac{1}{2}\frac{1+\frac{x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}+\frac{1}{2}\left(\sqrt{1+x^2}+x\frac{x}{\sqrt{1+x^2}}\right)=$$

$$=\frac{1}{2}\frac{\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}+\frac{1}{2}\frac{1+x^2+x^2}{\sqrt{1+x^2}}=$$

$$=\frac{1}{2}\frac{1}{\sqrt{1+x^2}}+\frac{1}{2}\frac{1+2x^2}{\sqrt{1+x^2}}=\frac{2+2x^2}{2\sqrt{1+x^2}}=\sqrt{1+x^2}$$

```
In[1]:= Integrate [ Sqrt[1+x^2], x]

                     2
         x Sqrt[1 + x ] + ArcSinh[x]
Out[1]= ----------------------------
                      2

In[2]:= TrigToExp[%]

                     2                         2
         x Sqrt[1 + x ]   Log[x + Sqrt[1 + x ]]
Out[2]= -------------- + ---------------------
               2                    2
```

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