```
> read "DRStrelba.m":
```

Apl. pŠíklad 1:

Izotermní vnitŠní difuze v porézním katalyzátoru je popsána diferenciální rovnicí

$y''+a/x\,y'=\varphi^2 y^n$ s okrajovou podmínkou y'(0)=0 a y(1)=1.

Parametr a charakterizuje tvar ļástice katalyzátoru a=0 pro desku, a=1pro váleļek, a=2 pro kuliļku, n je Šád reakce a ◖ Thieleho modul.

a) VyŠeģte tuto rovnici metodou stŠelby pro n=0, a=1 a ◖=1

b) VyŠeģte tuto rovnici metodou stŠelby pro n=1, a=2 a ◖=1;2;4

c) VyŠeģte tuto rovnici metodou stŠelby pro n=1, a=0 a ◖=1;2;4

a)

Definice parametrŢ diferenciální rovnice

```
> n:=0:
  aa:=1:
  phi:=1:
```

Definice pravé strany diferenciální rovnice

```
> vv:=piecewise(x=0,1/(1+aa)*phi^2*y1^n,-aa/x*y2 + phi^2*y1^n)
  ;
```

$$vv := \begin{cases} \frac{1}{2} & x = 0 \\ -\frac{y2}{x} + 1 & otherwise \end{cases} \qquad \textbf{(1.1)}$$

```
> f:=unapply(y2,x,y1,y2);
  g:=unapply(vv,x,y1,y2);
```

$$f := (x, y1, y2) \rightarrow y2$$

$$g := (x, y1, y2) \rightarrow piecewise\left(x = 0, \frac{1}{2}, -\frac{y2}{x} + 1\right) \qquad \textbf{(1.2)}$$

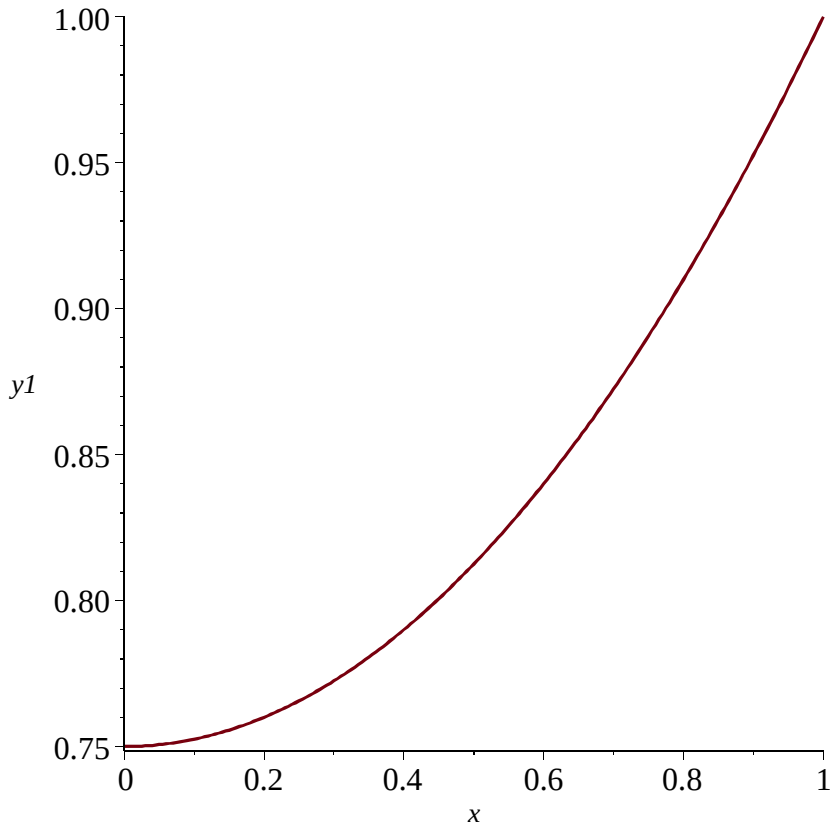
Definice parametrŢ metody

```
> a := 0:
  b := 1:
  alfa1 := 0:
  alfa2 := 1:
  beta1 := 1:
  beta2 := 0:
  gama1 := 0:
  gama2 := 1:
  eps := 0.1e-4:
  m := 10:
  h:=(b-a)/m:
  z0:=0.1;
  Lx := evalf([seq(a+(i-1)*h, i = 1 .. m+1)]):
```

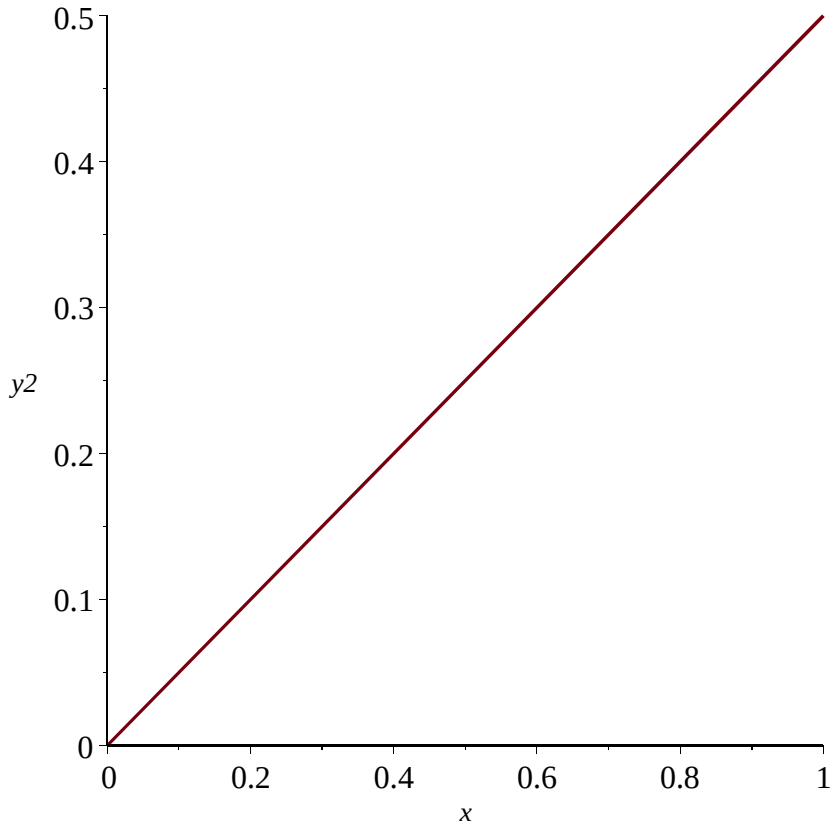
$$z0 := 0.1 \qquad \textbf{(1.3)}$$

```
> v := Strelba2(f, g, a, b, alfa1,alfa2,beta1,beta2,gama1,
  gama2, eps, z0, Lx):
iterace        zn           sn
---------------------------------------
    0     0.100000000
    1     0.750000000     0.650000000
    2     0.750000000     0.000000000
```

```
> # Graf funkce y1(x)
> v[1];
```




```
> # Graf funkce y2(x)
> v[2];
```




```
> # Tabulka hodnot funkce y1(x)
> linalg[matrix](v[3]);
```

0.	0.750000000000000
0.1000000000	0.752500000000000
0.2000000000	0.760000000000000
0.3000000000	0.772500000000000
0.4000000000	0.790000000000000
0.5000000000	0.812500000000000
0.6000000000	0.840000000000000
0.7000000000	0.872500000000000
0.8000000000	0.910000000000000
0.9000000000	0.952500000000000
1.	1.

(1.4)

```
> # Tabulka hodnot funkce y2(x)
> linalg[matrix](v[4]);
```

0.	0.
0.1000000000	0.0500000000000000
0.2000000000	0.100000000000000
0.3000000000	0.150000000000000
0.4000000000	0.200000000000000
0.5000000000	0.250000000000000
0.6000000000	0.300000000000000
0.7000000000	0.350000000000000
0.8000000000	0.400000000000000
0.9000000000	0.450000000000000
1.	0.500000000000000

(1.5)

b)

Definice parametrŢ diferenciální rovnice

```
> n:=1:
  aa:=2:
  phi:=1:
```

Definice pravé strany diferenciální rovnice

```
> vv:=piecewise(x=0,1/(1+aa)*phi^2*y1^n,-aa/x*y2 + phi^2*y1^n)
  ;
```

$$vv := \begin{cases} \frac{1}{3}\, y1 & x = 0 \\ -\frac{2\, y2}{x} + y1 & otherwise \end{cases} \qquad \textbf{(2.1)}$$

```
> f:=unapply(y2,x,y1,y2);
  g:=unapply(vv,x,y1,y2);
```

$$f := (x, y1, y2) \rightarrow y2$$

$$g := (x, y1, y2) \rightarrow piecewise\left(x = 0,\ \frac{1}{3}\, y1,\ -\frac{2\, y2}{x} + y1\right) \qquad \textbf{(2.2)}$$

Definice parametrŢ metody

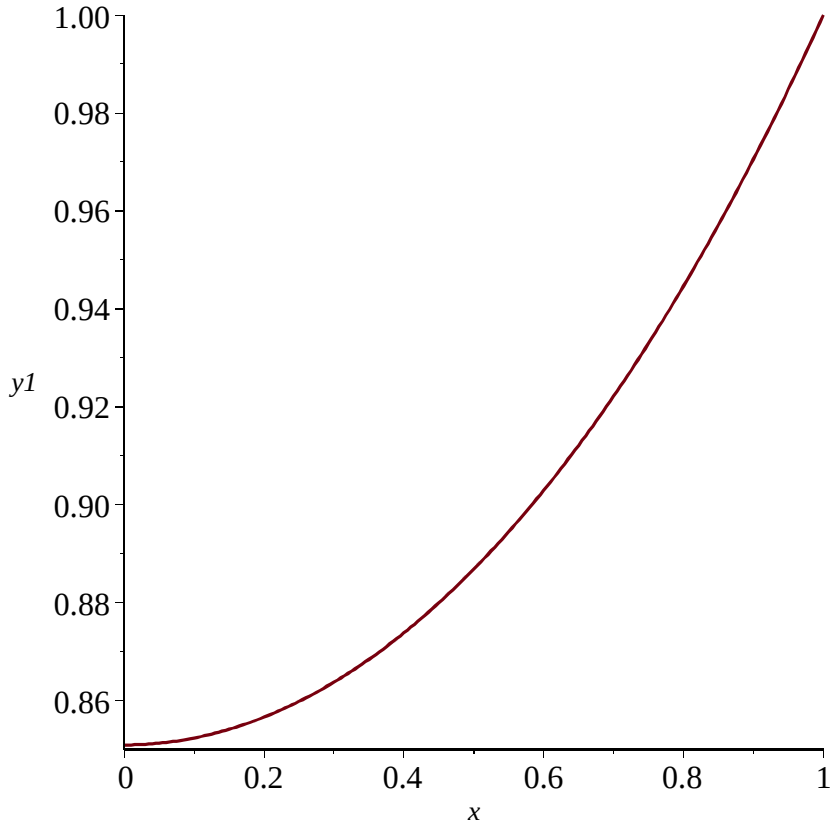
```
> a := 0:
  b := 1:
  alfa1 := 0:
  alfa2 := 1:
  beta1 := 1:
  beta2 := 0:
  gama1 := 0:
  gama2 := 1:
  eps := 0.1e-4:
  m := 10:
  h:=(b-a)/m:
  z0:=0.1;
  Lx := evalf([seq(a+(i-1)*h, i = 1 .. m+1)]):
```

$$z0 := 0.1 \qquad \textbf{(2.3)}$$

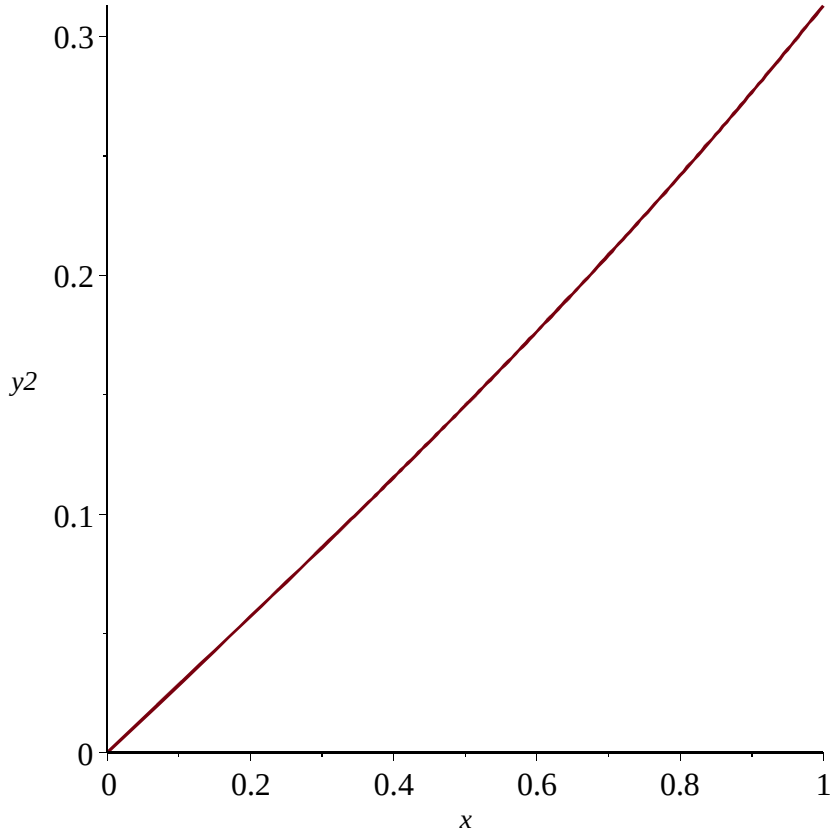
```
> v := Strelba2(f, g, a, b, alfa1,alfa2,beta1,beta2,gama1,
  gama2, eps, z0, Lx):
```

iterace	zn	sn
0	0.100000000	
1	0.850918111	0.750918111
2	0.850918111	0.000000000

```
> # Graf funkce y1(x)
> v[1];
```




```
> # Graf funkce y2(x)
> v[2];
```




```
> # Tabulka hodnot funkce y1(x)
> linalg[matrix](v[3]);
```

0.	0.850918110587666
0.1000000000	0.852337016819590
0.2000000000	0.856602255672785
0.3000000000	0.863739444297938
0.4000000000	0.873791484389313
0.5000000000	0.886818868002795
0.6000000000	0.902900113678745
0.7000000000	0.922132312850405
0.8000000000	0.944631838446477
0.9000000000	0.970535213591088
1.	1.00000000000000

(2.4)

```
> # Tabulka hodnot funkce y2(x)
> linalg[matrix](v[4]);
```

0.	0.
0.1000000000	0.0283923061708346
0.2000000000	0.0569550927297431
0.3000000000	0.0858600919170732
0.4000000000	0.115281441191533
0.5000000000	0.145396980992648
0.6000000000	0.176389503409356
0.7000000000	0.208448169391748
0.8000000000	0.241769734508640
0.9000000000	0.276559922360380
1.	0.313035265110230

(2.5)

Definice parametrŢ diferenciální rovnice

```
> n:=1:
  aa:=2:
  phi:=2:
```

Definice pravé strany diferenciální rovnice

```
> vv:=piecewise(x=0,1/(1+aa)*phi^2*y1^n,-aa/x*y2 + phi^2*y1^n)
  ;
```

$$vv := \begin{cases} \frac{4}{3}\, y1 & x = 0 \\ -\frac{2\, y2}{x} + 4\, y1 & otherwise \end{cases} \qquad \textbf{(2.6)}$$

```
> f:=unapply(y2,x,y1,y2);
  g:=unapply(vv,x,y1,y2);
```

$$f := (x, y1, y2) \rightarrow y2$$

$$g := (x, y1, y2) \rightarrow piecewise\left(x = 0,\ \frac{4}{3}\, y1,\ -\frac{2\, y2}{x} + 4\, y1\right) \qquad \textbf{(2.7)}$$

Definice parametrŢ metody

```
> a := 0:
  b := 1:
  alfa1 := 0:
  alfa2 := 1:
  beta1 := 1:
  beta2 := 0:
  gama1 := 0:
  gama2 := 1:
  eps := 0.1e-4:
  m := 10:
  h:=(b-a)/m:
  z0:=0.1;
  Lx := evalf([seq(a+(i-1)*h, i = 1 .. m+1)]):
```

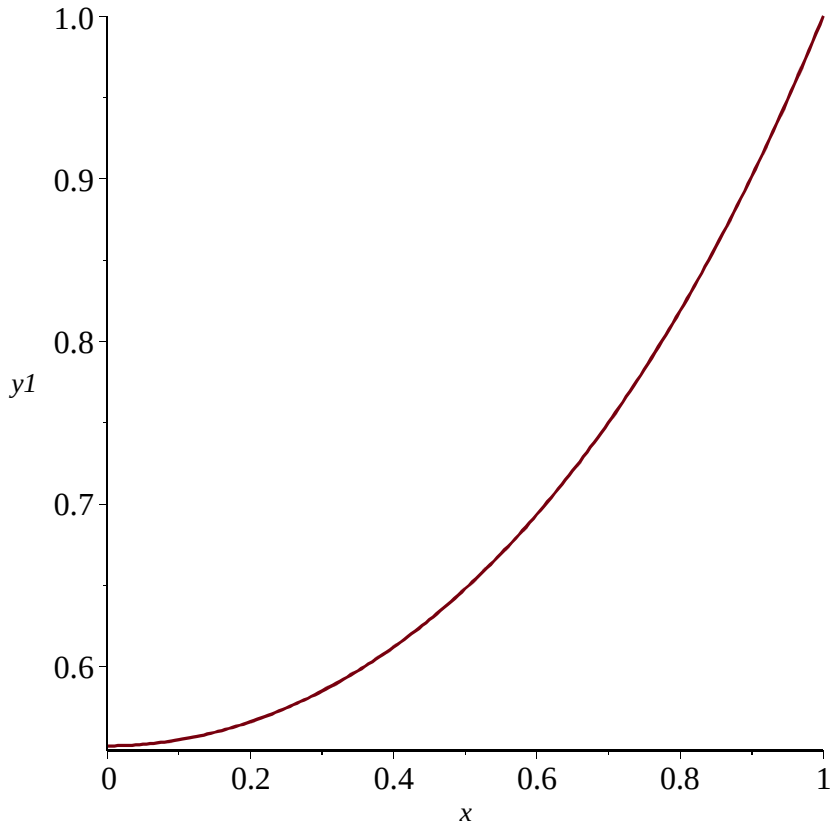
$$z0 := 0.1 \qquad \textbf{(2.8)}$$

```
> v := Strelba2(f, g, a, b, alfa1,alfa2,beta1,beta2,gama1,
  gama2, eps, z0, Lx):
iterace        zn           sn
-------------------------------------
    0     0.100000000
    1     0.551441071     0.451441071
```

```
    2      0.551441071      0.000000000
```

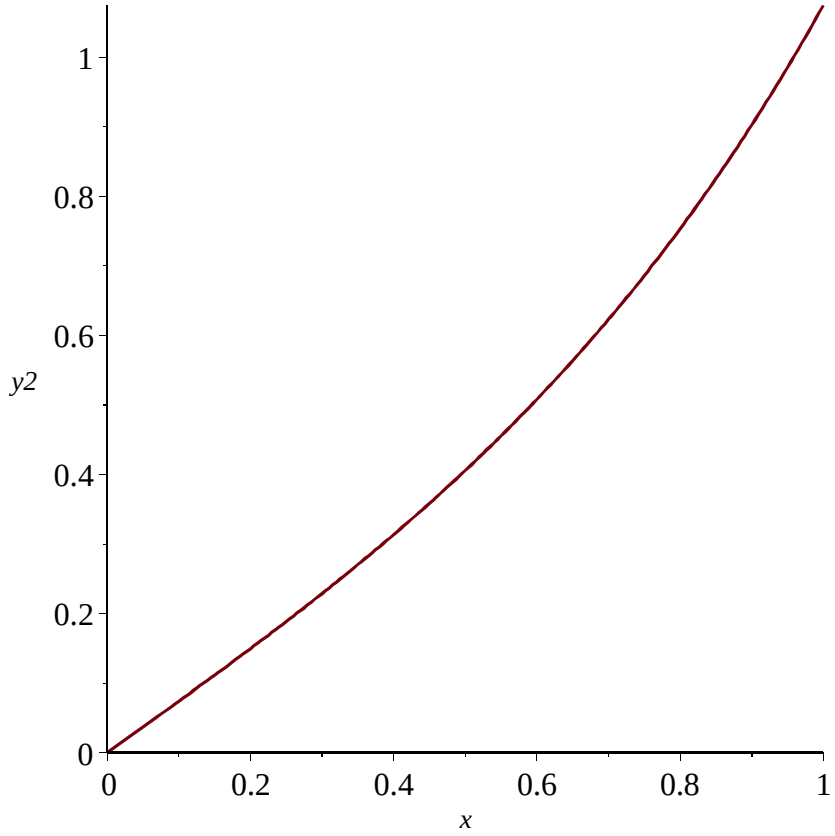
```
> # Graf funkce y1(x)
```

```
> v[1];
```




```
> # Graf funkce y2(x)
```

```
> v[2];
```




```
> # Tabulka hodnot funkce y1(x)
> linalg[matrix](v[3]);
```

0.	0.551441070571833
0.1000000000	0.555124704042958
0.2000000000	0.566264256504737
0.3000000000	0.585128225467607
0.4000000000	0.612172646593230
0.5000000000	0.648054219156632
0.6000000000	0.693649180559959
0.7000000000	0.750078651271825
0.8000000000	0.818741139447931
0.9000000000	0.901353267794138
1.	1.00000000000000

(2.9)

```
> # Tabulka hodnot funkce y2(x)
> linalg[matrix](v[4]);
```

0.	0.
0.1000000000	0.0738199996321619
0.2000000000	0.149417251152197
0.3000000000	0.228619933758951
0.4000000000	0.313359789189258
0.5000000000	0.405727630782555
0.6000000000	0.508034426563359
0.7000000000	0.622878703061211
0.8000000000	0.753223279655716
0.9000000000	0.902483425175413
1.	1.07462928140235

(2.10)

Definice parametrŢ diferenciální rovnice

```
> n:=1:
  aa:=2:
  phi:=4:
```

Definice pravé strany diferenciální rovnice

```
> vv:=piecewise(x=0,1/(1+aa)*phi^2*y1^n,-aa/x*y2 + phi^2*y1^n)
  ;
```

$$vv := \begin{cases} \frac{16}{3}\, y1 & x = 0 \\ -\frac{2\, y2}{x} + 16\, y1 & otherwise \end{cases} \qquad \textbf{(2.11)}$$

```
> f:=unapply(y2,x,y1,y2);
  g:=unapply(vv,x,y1,y2);
```

$$f := (x, y1, y2) \rightarrow y2$$

$$g := (x, y1, y2) \rightarrow piecewise\left(x = 0,\ \frac{16}{3}\, y1,\ -\frac{2\, y2}{x} + 16\, y1\right) \qquad \textbf{(2.12)}$$

Definice parametrŢ metody

```
> a := 0:
  b := 1:
  alfa1 := 0:
  alfa2 := 1:
  beta1 := 1:
  beta2 := 0:
  gama1 := 0:
  gama2 := 1:
  eps := 0.1e-4:
  m := 10:
  h:=(b-a)/m:
  z0:=0.1;
  Lx := evalf([seq(a+(i-1)*h, i = 1 .. m+1)]):
```

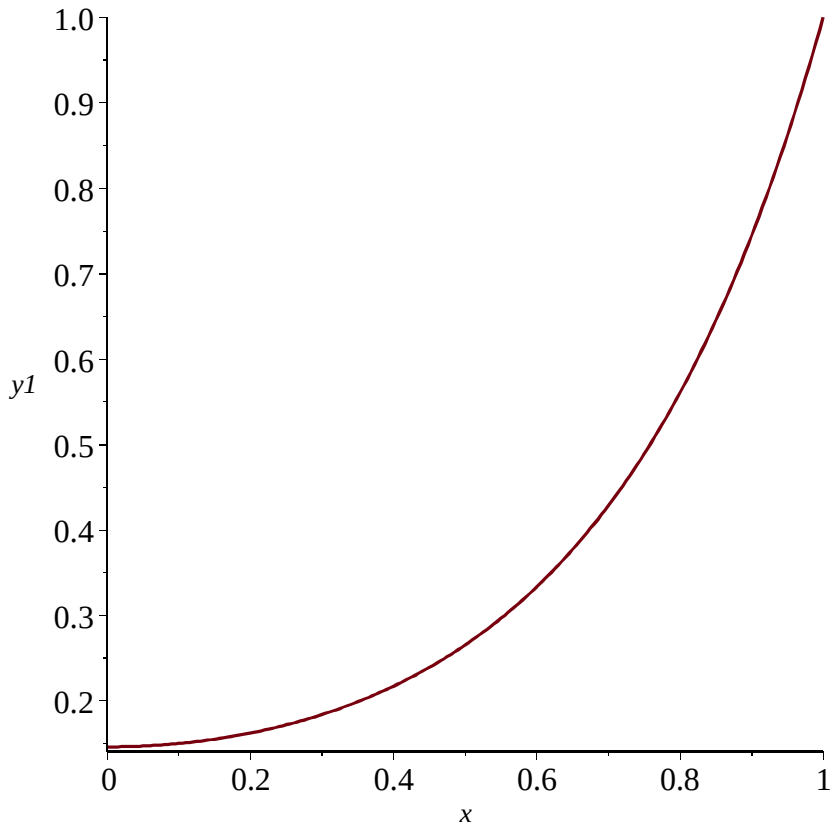
$$z0 := 0.1 \qquad \textbf{(2.13)}$$

```
> v := Strelba2(f, g, a, b, alfa1,alfa2,beta1,beta2,gama1,
  gama2, eps, z0, Lx):
iterace          zn              sn
-------------------------------------
    0      0.100000000
```

```
    1      0.146574277      0.046574277
    2      0.146574277      0.000000000
```

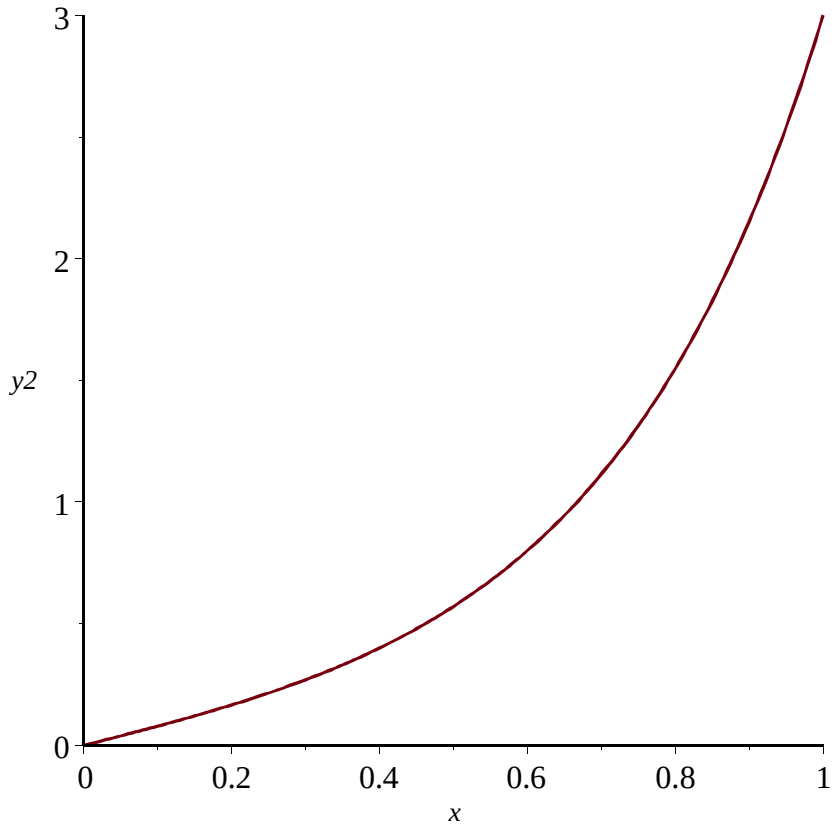
```
> # Graf funkce y1(x)
```

```
> v[1];
```




```
> # Graf funkce y2(x)
```

```
> v[2];
```




```
> # Tabulka hodnot funkce y1(x)
> linalg[matrix](v[3]);
```

0.	0.146574276721063
0.1000000000	0.150514312856873
0.2000000000	0.162716866067825
0.3000000000	0.184373511254641
0.4000000000	0.217623235581613
0.5000000000	0.265802244679787
0.6000000000	0.333836939757901
0.7000000000	0.428830239277032
0.8000000000	0.560916168905884
0.9000000000	0.744493736558281
1.	1.00000000000000

(2.14)

```
> # Tabulka hodnot funkce y2(x)
> linalg[matrix](v[4]);
```

0.	0.
0.1000000000	0.0794308818006772
0.2000000000	0.166583472780227
0.3000000000	0.270073395556438
0.4000000000	0.400416892071411
0.5000000000	0.571277769663465
0.6000000000	0.801114291301337
0.7000000000	1.11543926857212
0.8000000000	1.54998767123255
0.9000000000	2.15520942488935
1.	3.00268437876257

(2.15)

c)

Definice parametrŢ diferenciální rovnice

```
> n:=1:
  aa:=0:
  phi:=1:
```

Definice pravé strany diferenciální rovnice

```
> vv:= phi^2*y1^n;
```

$$vv := y1 \qquad \textbf{(3.1)}$$

```
> f:=unapply(y2,x,y1,y2);
  g:=unapply(vv,x,y1,y2);
```

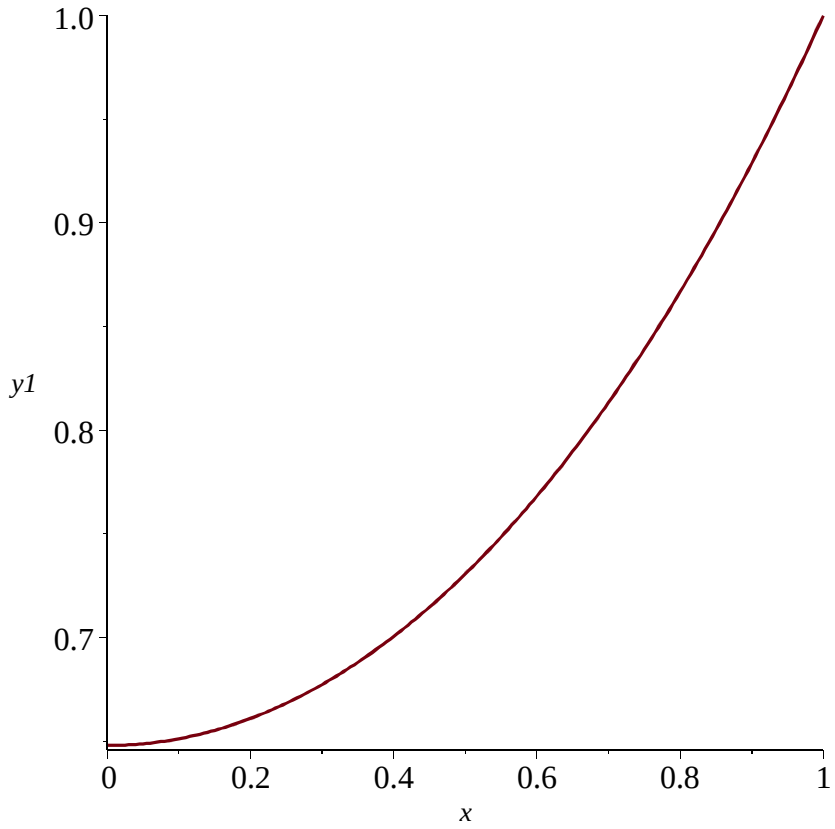
$$f := (x, y1, y2) \rightarrow y2$$
$$g := (x, y1, y2) \rightarrow y1 \qquad \textbf{(3.2)}$$

Definice parametrŢ metody

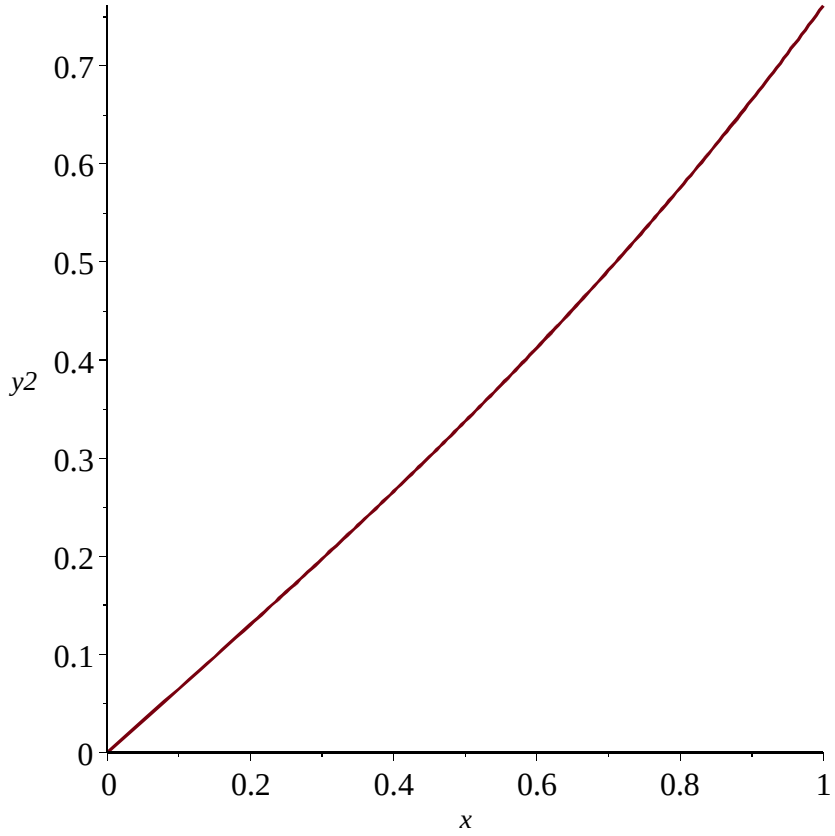
```
> a := 0:
  b := 1:
  alfa1 := 0:
  alfa2 := 1:
  beta1 := 1:
  beta2 := 0:
  gama1 := 0:
  gama2 := 1:
  eps := 0.1e-4:
  m := 10:
  h:=(b-a)/m:
  z0:=0.1;
  Lx := evalf([seq(a+(i-1)*h, i = 1 .. m+1)]):
```

$$z0 := 0.1 \qquad \textbf{(3.3)}$$

```
> v := Strelba2(f, g, a, b, alfa1,alfa2,beta1,beta2,gama1,
  gama2, eps, z0, Lx):
iterace        zn           sn
-------------------------------------
    0     0.100000000
    1     0.648054317     0.548054317
    2     0.648054317     0.000000000
> # Graf funkce y1(x)
> v[1];
```




```
> # Graf funkce y2(x)
> v[2];
```




```
> # Tabulka hodnot funkce y1(x)
> linalg[matrix](v[3]);
```

0.	0.648054316600890
0.1000000000	0.651297287780308
0.2000000000	0.661058660894575
0.3000000000	0.677436133011274
0.4000000000	0.700593608226644
0.5000000000	0.730762859393532
0.6000000000	0.768245841471974
0.7000000000	0.813417659858372
0.8000000000	0.866730462001918
0.9000000000	0.928717773057839
1.	1.00000000000000

(3.4)

```
> # Tabulka hodnot funkce y2(x)
> linalg[matrix](v[4]);
```

0.	0.
0.1000000000	0.0649134955668701
0.2000000000	0.130476670570728
0.3000000000	0.197345703019773
0.4000000000	0.266189816595520
0.5000000000	0.337698064744230
0.6000000000	0.412586119478881
0.7000000000	0.491603425750418
0.8000000000	0.575540921111694
0.9000000000	0.665238574653541
1.	0.761594171751355

(3.5)

Definice parametrŢ diferenciální rovnice

```
> n:=1:
  aa:=0:
  phi:=2:
```

Definice pravé strany diferenciální rovnice

```
> vv:= phi^2*y1^n;
```

$$vv := 4\,y1 \tag{3.6}$$

```
> f:=unapply(y2,x,y1,y2);
  g:=unapply(vv,x,y1,y2);
```

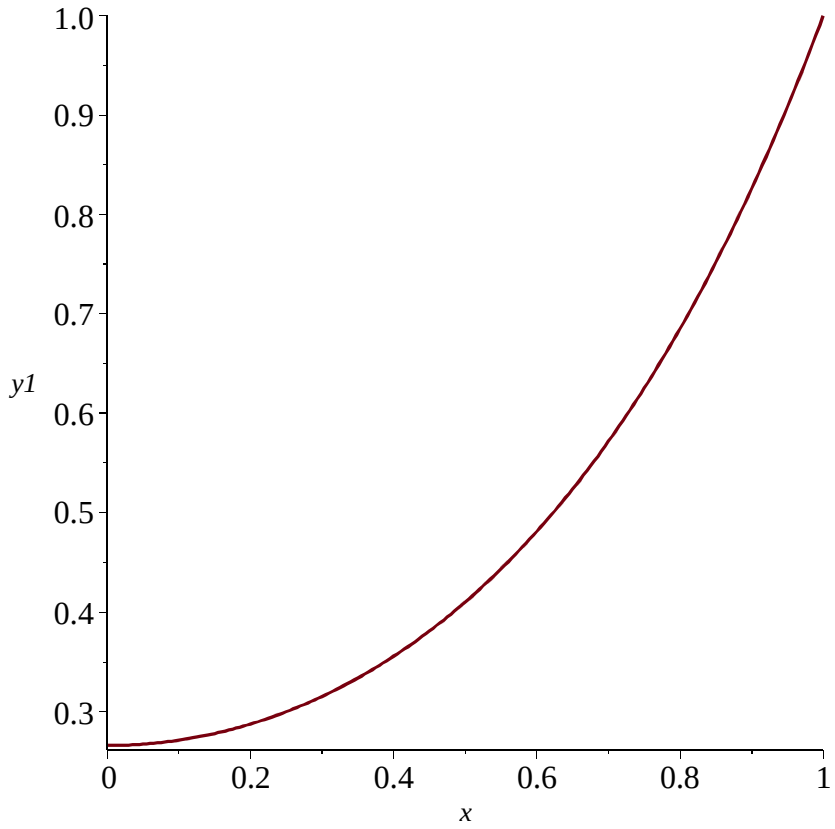
$$f := (x, y1, y2) \rightarrow y2$$
$$g := (x, y1, y2) \rightarrow 4\,y1 \tag{3.7}$$

Definice parametrŢ metody

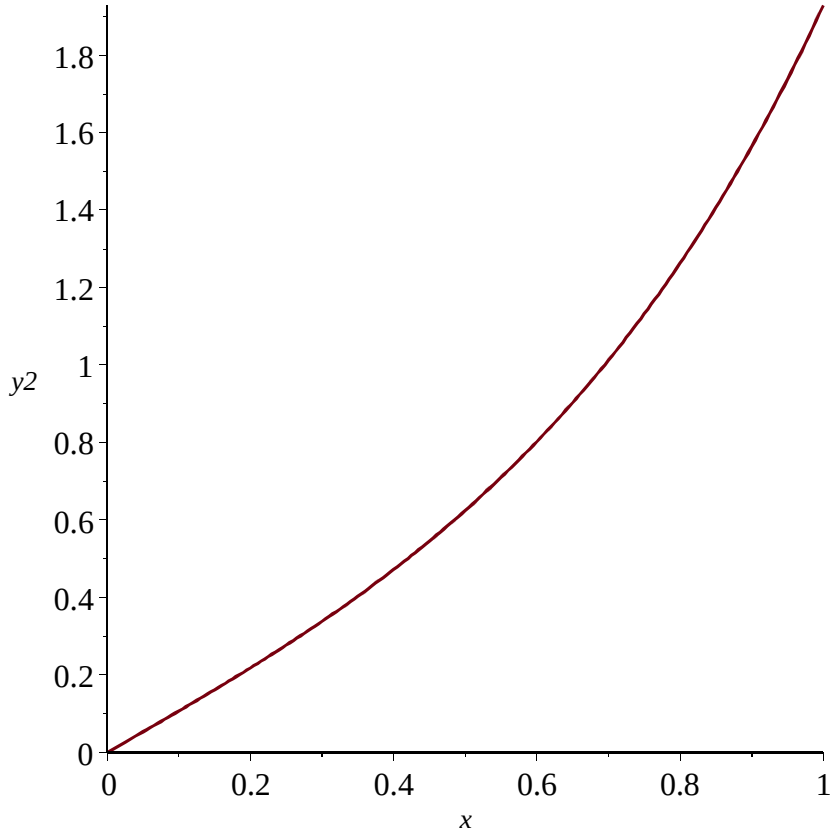
```
> a := 0:
  b := 1:
  alfa1 := 0:
  alfa2 := 1:
  beta1 := 1:
  beta2 := 0:
  gama1 := 0:
  gama2 := 1:
  eps := 0.1e-4:
  m := 10:
  h:=(b-a)/m:
  z0:=0.1;
  Lx := evalf([seq(a+(i-1)*h, i = 1 .. m+1)]):
```

$$z0 := 0.1 \tag{3.8}$$

```
> v := Strelba2(f, g, a, b, alfa1,alfa2,beta1,beta2,gama1,
  gama2, eps, z0, Lx):
iterace        zn            sn
-------------------------------------
    0     0.100000000
    1     0.265802275     0.165802275
    2     0.265802275     0.000000000
> # Graf funkce y1(x)
> v[1];
```




```
> # Graf funkce y2(x)
> v[2];
```




```
> # Tabulka hodnot funkce y1(x)
> linalg[matrix](v[3]);
```

$$\begin{bmatrix} 0. & 0.265802274834478 \\ 0.1000000000 & 0.271136063262046 \\ 0.2000000000 & 0.287351494358305 \\ 0.3000000000 & 0.315099345557039 \\ 0.4000000000 & 0.355493235266235 \\ 0.5000000000 & 0.410154315983149 \\ 0.6000000000 & 0.481276334401872 \\ 0.7000000000 & 0.571713660101080 \\ 0.8000000000 & 0.685095852168439 \\ 0.9000000000 & 0.825973330804636 \\ 1. & 1.00000000000000 \end{bmatrix} \quad \textbf{(3.9)}$$

```
> # Tabulka hodnot funkce y2(x)
> linalg[matrix](v[4]);
```

0.	0.
0.1000000000	0.107031135381465
0.2000000000	0.218357815649302
0.3000000000	0.338447946106814
0.4000000000	0.472121163883811
0.5000000000	0.624742263684027
0.6000000000	0.802436478868445
0.7000000000	1.01233528112441
0.8000000000	1.26286262749709
0.9000000000	1.56407305041994
1.	1.92805517525232

(3.10)

Definice parametrŢ diferenciální rovnice

```
> n:=1:
  aa:=0:
  phi:=4:
```

Definice pravé strany diferenciální rovnice

```
> vv:= phi^2*y1^n;
```

$$vv := 16\, y1 \qquad \textbf{(3.11)}$$

```
> f:=unapply(y2,x,y1,y2);
  g:=unapply(vv,x,y1,y2);
```

$$f := (x, y1, y2) \rightarrow y2$$

$$g := (x, y1, y2) \rightarrow 16\, y1 \qquad \textbf{(3.12)}$$

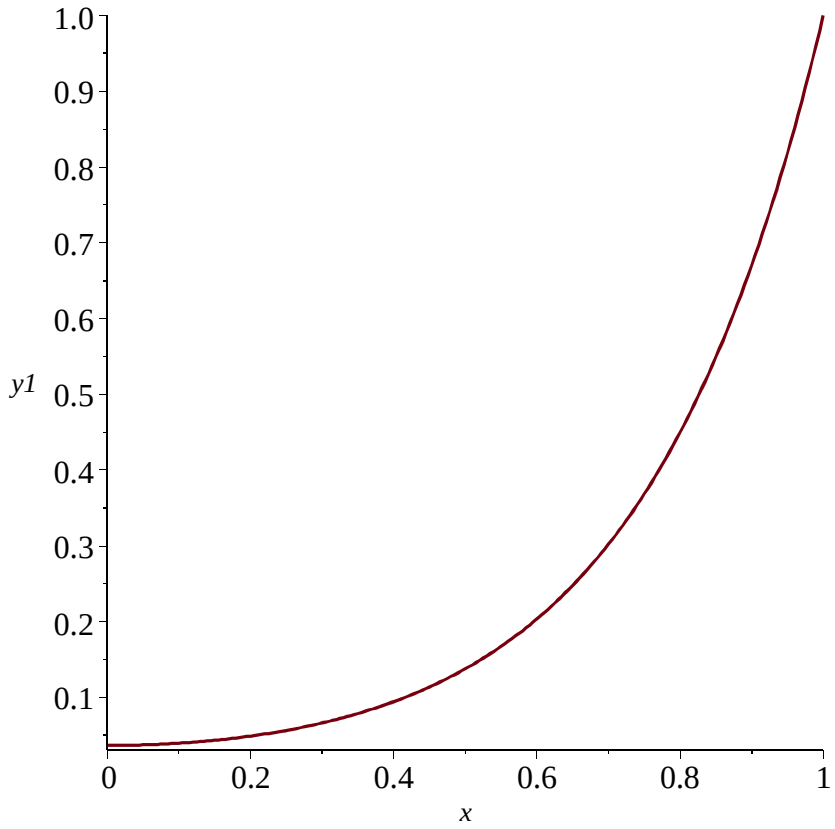
Definice parametrŢ metody

```
> a := 0:
  b := 1:
  alfa1 := 0:
  alfa2 := 1:
  beta1 := 1:
  beta2 := 0:
  gama1 := 0:
  gama2 := 1:
  eps := 0.1e-4:
  m := 10:
  h:=(b-a)/m:
  z0:=0.1;
  Lx := evalf([seq(a+(i-1)*h, i = 1 .. m+1)]):
```

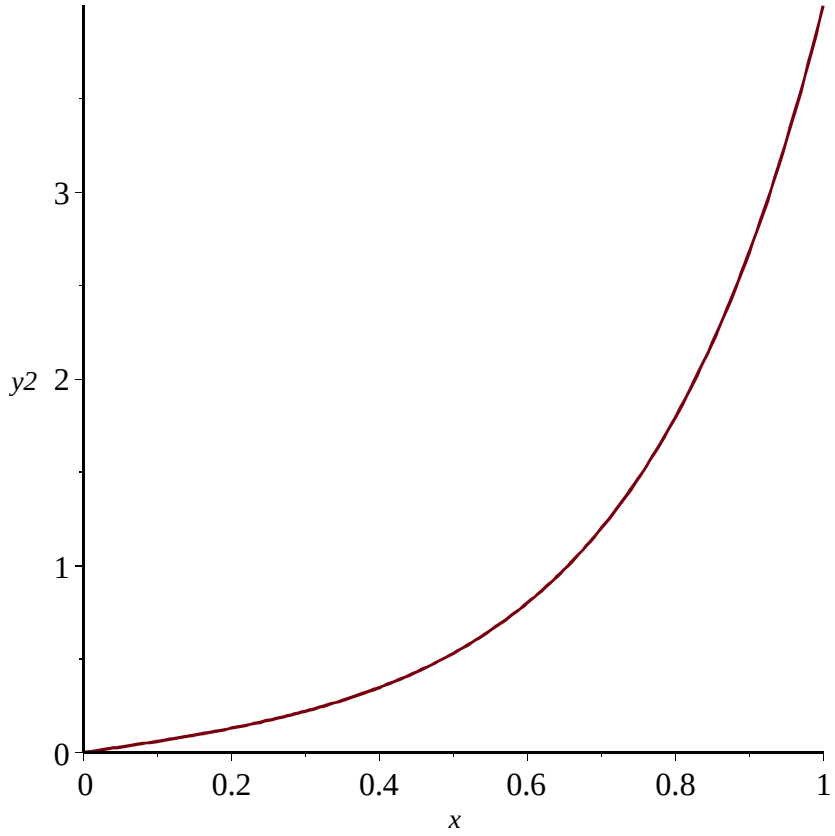
$$z0 := 0.1 \qquad \textbf{(3.13)}$$

```
> v := Strelba2(f, g, a, b, alfa1,alfa2,beta1,beta2,gama1,
  gama2, eps, z0, Lx):
iterace        zn            sn
-------------------------------------
    0     0.100000000
    1     0.036619013     0.063380987
    2     0.036619013     0.000000000
```

```
> # Graf funkce y1(x)
> v[1];
```




```
> # Graf funkce y2(x)
> v[2];
```




```
> # Tabulka hodnot funkce y1(x)
> linalg[matrix](v[3]);
```

0.	0.0366190130410772
0.1000000000	0.0395878029557592
0.2000000000	0.0489755466152499
0.3000000000	0.0663044160655094
0.4000000000	0.0943841918585146
0.5000000000	0.137767862999034
0.6000000000	0.203489863951059
0.7000000000	0.302206674464586
0.8000000000	0.449924694015984
0.9000000000	0.670595601652913
1.	1.00000000000000

(3.14)

```
> # Tabulka hodnot funkce y2(x)
> linalg[matrix](v[4]);
```

$$\left[\begin{array}{cc} 0. & 0. \\ 0.1000000000 & 0.0601653804997799 \\ 0.2000000000 & 0.130086258900137 \\ 0.3000000000 & 0.221099933374118 \\ 0.4000000000 & 0.347963769477879 \\ 0.5000000000 & 0.531248079770433 \\ 0.6000000000 & 0.800671453227637 \\ 0.7000000000 & 1.19991948459535 \\ 0.8000000000 & 1.79372809328149 \\ 0.9000000000 & 2.67838013869573 \\ 1. & 3.99731719980291 \end{array}\right] \quad \textbf{(3.15)}$$