```
> read "DRStrelba.m":
```

Apl. pŠíklad 6:

Jednoduchý model chování sloupce plasmy stlaļované záŠením odvodil Troesch ve tvaru nelineární okrajové úlohy

$y''=\alpha \sinh \check{U}y$

y(0)=0 a y(1)=1.

Pouģijte parametry

1) $\check{U}=0,8$;
2) $\check{U}=1$;
3) $\alpha=2$;
4) $\alpha=5$;
5) $\alpha=10$;
6) $\alpha=20$.

1)

Definice parametrŢ diferenciální rovnice

```
> alpha:=0.8:
```

Definice pravé strany diferenciální rovnice

```
> f:=unapply(y2,x,y1,y2);
  g:=unapply(alpha*sinh(alpha*y1),x,y1,y2);
```

$$f := (x, y1, y2) \rightarrow y2$$

$$g := (x, y1, y2) \rightarrow 0.8 \sinh(0.8\, y1) \qquad \textbf{(1.1)}$$

Definice parametrŢ metody

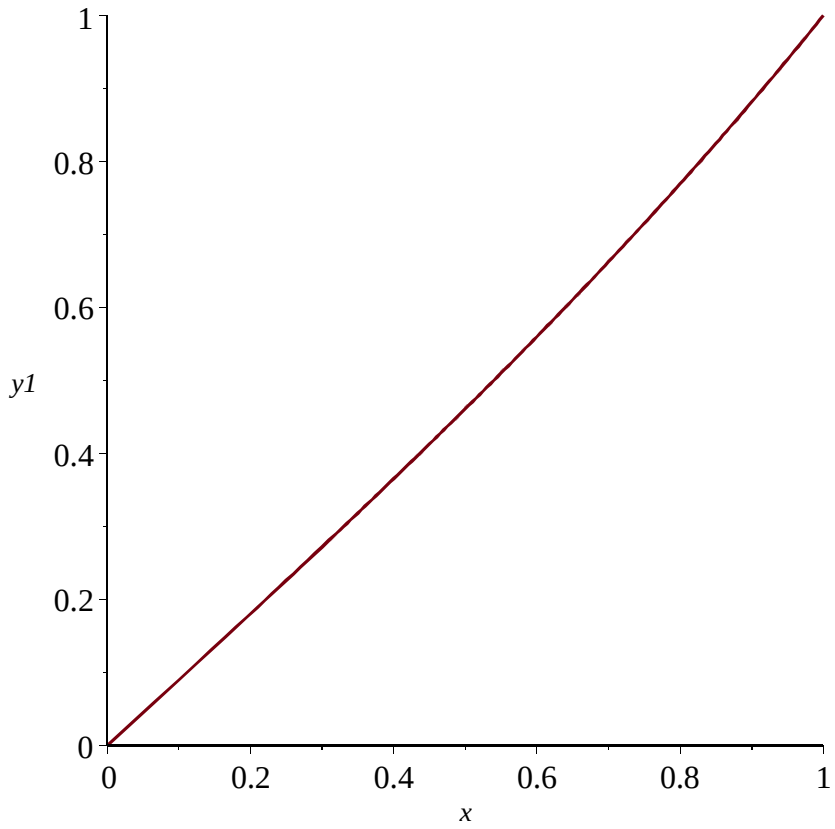
```
> a := 0:
  b := 1:
  alfa1 := 1:
  alfa2 := 1:
  beta1 := 0:
  beta2 := 0:
  gama1 := 0:
  gama2 := 1:
  eps := 0.1e-5:
  m := 10:
  h:=(b-a)/m:
  z0:=1.0;
  Lx := evalf([seq(a+(i-1)*h, i = 1 .. m+1)]):
```

$$z0 := 1.0 \qquad \textbf{(1.2)}$$

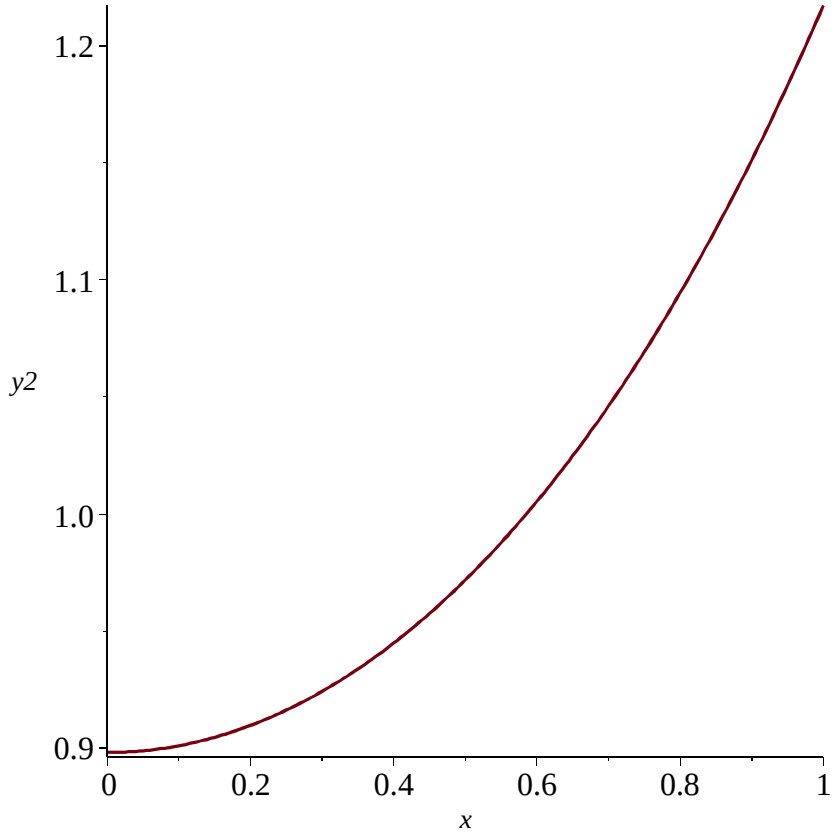
```
> v := Strelba2(f, g, a, b, alfa1,alfa2,beta1,beta2,gama1,
  gama2, eps, z0, Lx):
```

```
iterace        zn            sn
-------------------------------------
    0     1.000000000
    1     0.898230640     0.101769360
    2     0.898115225     0.000115416
    3     0.898115225     0.000000000
```

```
> # Graf funkce y1(x)
> v[1];
```




```
> # Graf funkce y2(x)
> v[2];
```




```
> # Tabulka hodnot funkce y1(x)
> linalg[matrix](v[3]);
```

0.	0.
0.1000000000	0.0899073767223764
0.2000000000	0.180391222339103
0.3000000000	0.272034706911292
0.4000000000	0.365434752933417
0.5000000000	0.461209509753983
0.6000000000	0.560006643694308
0.7000000000	0.662513151781499
0.8000000000	0.769466332185656
0.9000000000	0.881667765094804
1.	1.00000000015638

(1.3)

```
> # Tabulka hodnot funkce y2(x)
> linalg[matrix](v[4]);
```

0.	0.898115224651079
0.1000000000	0.900991964134545
0.2000000000	0.909655632645352
0.3000000000	0.924207773839923
0.4000000000	0.944822018969223
0.5000000000	0.971750771550474
0.6000000000	1.00533536157729
0.7000000000	1.04602105549658
0.8000000000	1.09437599130267
0.9000000000	1.15112017058405
1.	1.21716092599577

(1.4)

2)

Definice parametrŢ diferenciální rovnice

```
> alpha:=1.0:
```

Definice pravé strany diferenciální rovnice

```
> f:=unapply(y2,x,y1,y2);
  g:=unapply(alpha*sinh(alpha*y1),x,y1,y2);
```

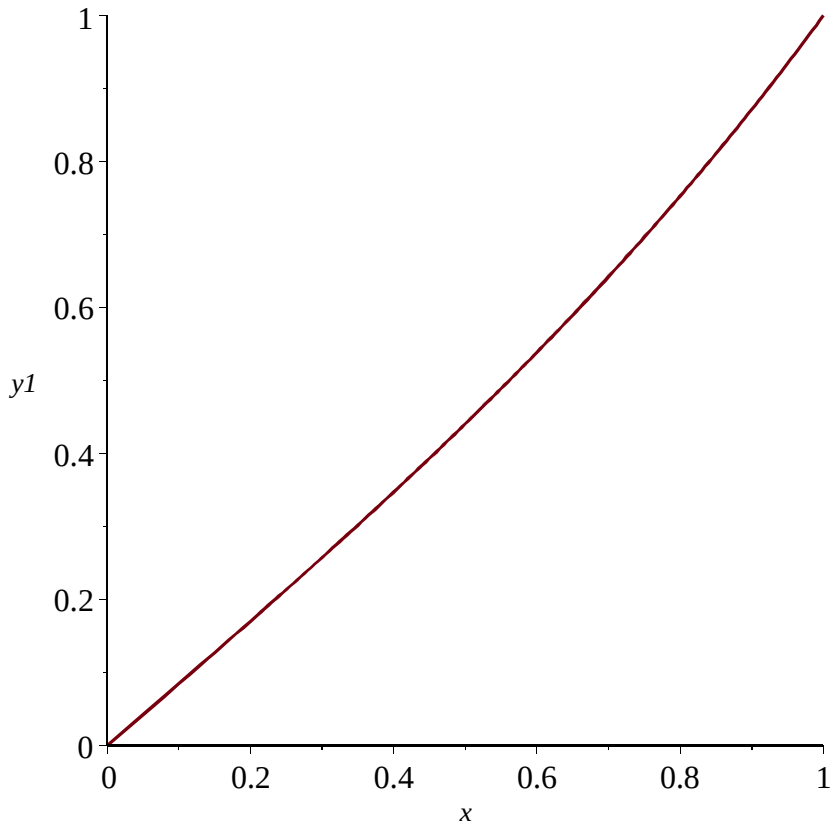
$$f := (x, y1, y2) \rightarrow y2$$
$$g := (x, y1, y2) \rightarrow 1.0 \sinh(1.0\, y1) \qquad \textbf{(2.1)}$$

Definice parametrŢ metody

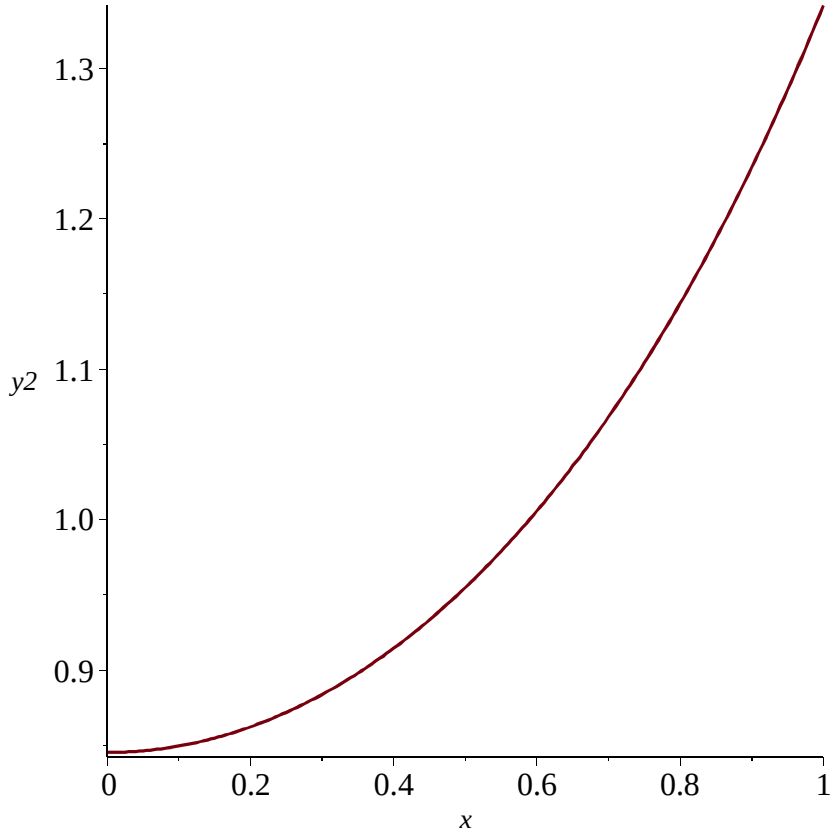
```
> a := 0:
  b := 1:
  alfa1 := 1:
  alfa2 := 1:
  beta1 := 0:
  beta2 := 0:
  gama1 := 0:
  gama2 := 1:
  eps := 0.1e-5:
  m := 10:
  h:=(b-a)/m:
  z0:=1.0;
  Lx := evalf([seq(a+(i-1)*h, i = 1 .. m+1)]):
```

$$z0 := 1.0 \qquad \textbf{(2.2)}$$

```
> v := Strelba2(f, g, a, b, alfa1,alfa2,beta1,beta2,gama1,
  gama2, eps, z0, Lx):
iterace          zn              sn
-------------------------------------
    0      1.000000000
    1      0.845887660      0.154112340
    2      0.845202726      0.000684933
    3      0.845202715      0.000000012
> # Graf funkce y1(x)
> v[1];
```




```
> # Graf funkce y2(x)
> v[2];
```




```
> # Tabulka hodnot funkce y1(x)
> linalg[matrix](v[3]);
```

0.	0.
0.1000000000	0.0846612602355925
0.2000000000	0.170171367390557
0.3000000000	0.257393930716095
0.4000000000	0.347222878080238
0.5000000000	0.440599849913693
0.6000000000	0.538534422951941
0.7000000000	0.642128649642225
0.8000000000	0.752608128873164
0.9000000000	0.871362541010301
1.	1.00000001400471

(2.3)

```
> # Tabulka hodnot funkce y2(x)
> linalg[matrix](v[4]);
```

0.	0.845202726493308
0.1000000000	0.849434785170620
0.2000000000	0.862204086056383
0.3000000000	0.883734063381229
0.4000000000	0.914411035575732
0.5000000000	0.954807138215576
0.6000000000	1.00571588316724
0.7000000000	1.06820454697818
0.8000000000	1.14369234817081
0.9000000000	1.23406393444513
1.	1.34183780032940

(2.4)

3)

Definice parametrŢ diferenciální rovnice

```
> alpha:=2.0:
```

Definice pravé strany diferenciální rovnice

```
> f:=unapply(y2,x,y1,y2);
  g:=unapply(alpha*sinh(alpha*y1),x,y1,y2);
```

$$f := (x, y1, y2) \rightarrow y2$$

$$g := (x, y1, y2) \rightarrow 2.0 \sinh(2.0\, y1) \qquad \textbf{(3.1)}$$

Definice parametrŢ metody

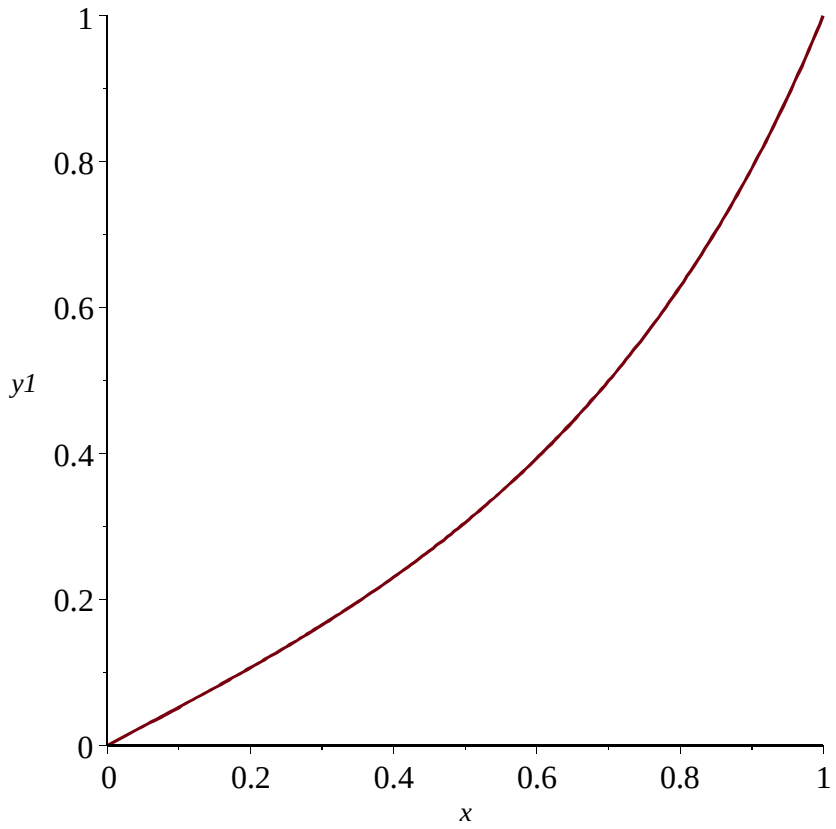
```
> a := 0:
  b := 1:
  alfa1 := 1:
  alfa2 := 1:
  beta1 := 0:
  beta2 := 0:
  gama1 := 0:
  gama2 := 1:
  eps := 0.1e-5:
  m := 10:
  h:=(b-a)/m:
  z0:=1.0;
  Lx := evalf([seq(a+(i-1)*h, i = 1 .. m+1)]):
```

$$z0 := 1.0 \qquad \textbf{(3.2)}$$

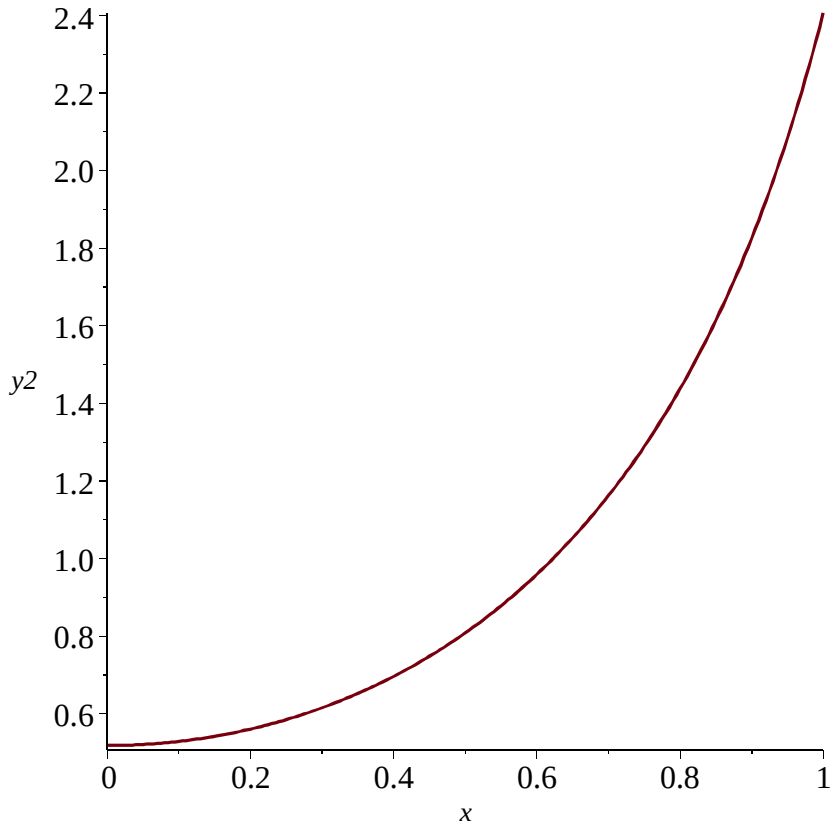
```
> v := Strelba2(f, g, a, b, alfa1,alfa2,beta1,beta2,gama1,
  gama2, eps, z0, Lx):
```

```
iterace        zn           sn
-------------------------------------
    0     1.000000000
    1     0.728781872     0.271218128
    2     0.544210759     0.184571113
    3     0.518902552     0.025308207
    4     0.518621302     0.000281250
    5     0.518621269     0.000000033
```

```
> # Graf funkce y1(x)
> v[1];
```




```
> # Graf funkce y2(x)
> v[2];
```




```
> # Tabulka hodnot funkce y1(x)
> linalg[matrix](v[3]);
```

0.	0.
0.1000000000	0.0522087623148978
0.2000000000	0.106518665711823
0.3000000000	0.165140850437268
0.4000000000	0.230521764618809
0.5000000000	0.305504758282769
0.6000000000	0.393563314188400
0.7000000000	0.499173013628892
0.8000000000	0.628465158556408
0.9000000000	0.790494063618935
1.	1.00000007125158

(3.3)

```
> # Tabulka hodnot funkce y2(x)
> linalg[matrix](v[4]);
```

0.	0.518621301904963
0.1000000000	0.529037788940532
0.2000000000	0.560825151501522
0.3000000000	0.615669780675774
0.4000000000	0.696649975962079
0.5000000000	0.808740342362373
0.6000000000	0.959787950551785
0.7000000000	1.16243099716460
0.8000000000	1.43809248950859
0.9000000000	1.82605985849873
1.	2.40693985200622

(3.4)

4)

Definice parametrŢ diferenciální rovnice

```
> alpha:=5.0:
```

Definice pravé strany diferenciální rovnice

```
> f:=unapply(y2,x,y1,y2);
  g:=unapply(alpha*sinh(alpha*y1),x,y1,y2);
```

$$f := (x, y1, y2) \rightarrow y2$$

$$g := (x, y1, y2) \rightarrow 5.0 \sinh(5.0\, y1) \qquad \textbf{(4.1)}$$

Definice parametrŢ metody

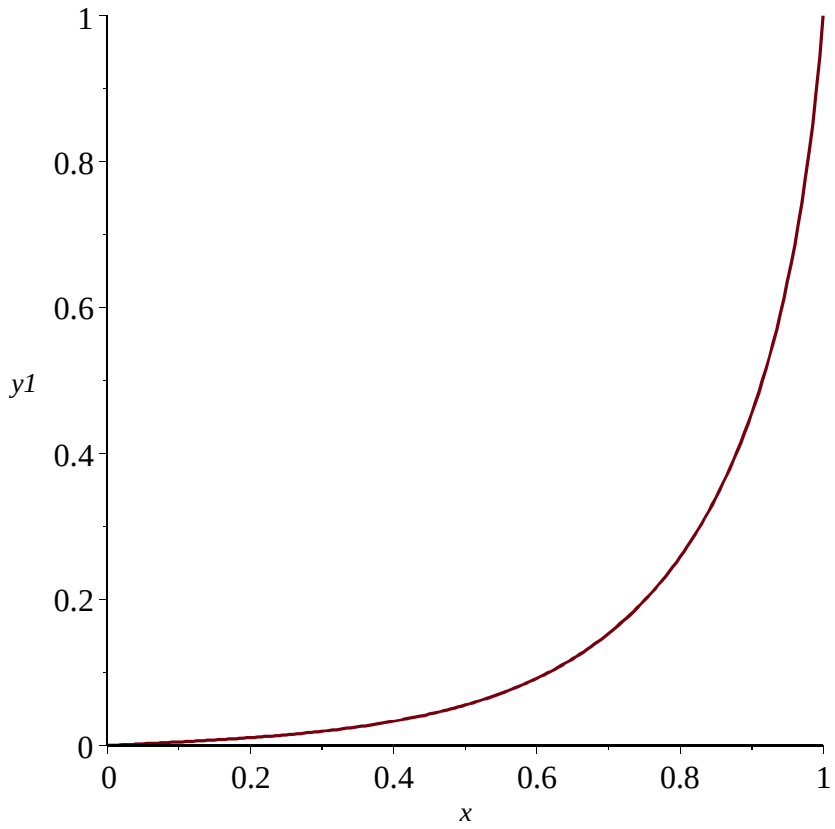
```
> a := 0:
  b := 1:
  alfa1 := 1:
  alfa2 := 1:
  beta1 := 0:
  beta2 := 0:
  gama1 := 0:
  gama2 := 1:
  eps := 0.1e-5:
  m := 10:
  h:=(b-a)/m:
  z0:=0.05;
  Lx := evalf([seq(a+(i-1)*h, i = 1 .. m+1)]):
```

$$z0 := 0.05 \qquad \textbf{(4.2)}$$

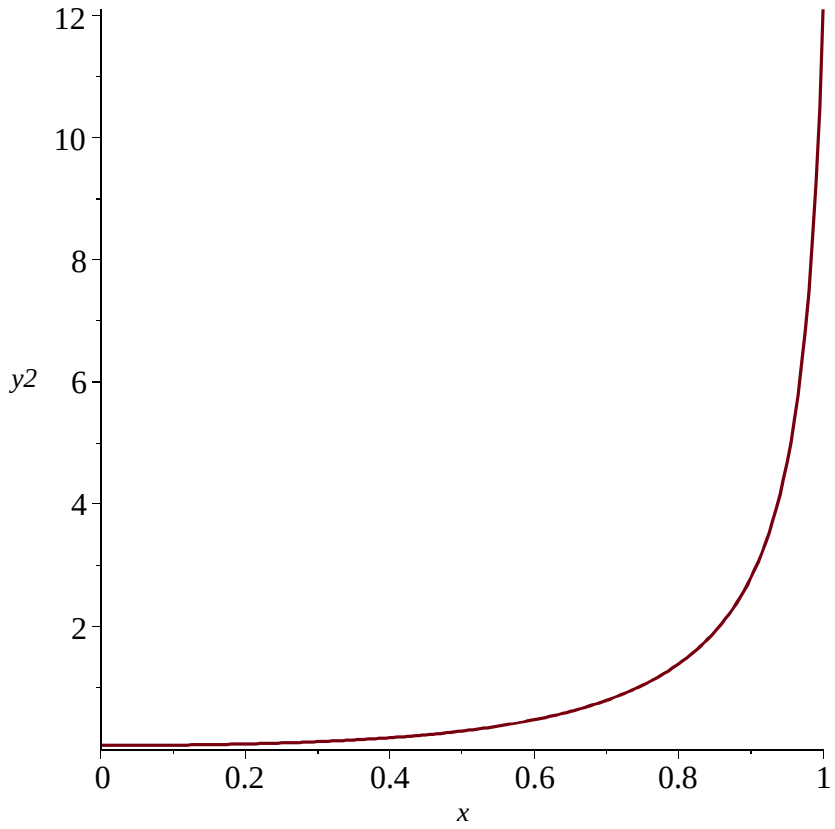
```
> v := Strelba2(f, g, a, b, alfa1,alfa2,beta1,beta2,gama1,
  gama2, eps, z0, Lx):
```

```
iterace          zn              sn
-------------------------------------
    0      0.050000000
    1      0.047063300      0.002936700
    2      0.045854130      0.001209170
    3      0.045751091      0.000103039
    4      0.045750486      0.000000605
```

```
> # Graf funkce y1(x)
> v[1];
```




```
> # Graf funkce y2(x)
> v[2];
```




```
> # Tabulka hodnot funkce y1(x)
> linalg[matrix](v[3]);
```

$$\begin{bmatrix} 0. & 0. \\ 0.1000000000 & 0.00476814088208659 \\ 0.2000000000 & 0.0107535538817829 \\ 0.3000000000 & 0.0194855460355271 \\ 0.4000000000 & 0.0332009374426686 \\ 0.5000000000 & 0.0554381398878860 \\ 0.6000000000 & 0.0920456073761345 \\ 0.7000000000 & 0.153163474234254 \\ 0.8000000000 & 0.258220147719482 \\ 0.9000000000 & 0.455067425713775 \\ 1. & 1.00003196668272 \end{bmatrix} \quad \textbf{(4.3)}$$

```
> # Tabulka hodnot funkce y2(x)
> linalg[matrix](v[4]);
```

0.	0.0457510906212831
0.1000000000	0.0515903880754758
0.2000000000	0.0706033367268117
0.3000000000	0.107670026866120
0.4000000000	0.172377665123152
0.5000000000	0.281817443073450
0.6000000000	0.466549172840872
0.7000000000	0.786001542046685
0.8000000000	1.38341919899707
0.9000000000	2.79929526422556
1.	12.1014748928741

(4.4)

5)

Definice parametrŢ diferenciální rovnice

```
> alpha:=10.0:
```

Definice pravé strany diferenciální rovnice

```
> f:=unapply(y2,x,y1,y2);
  g:=unapply(alpha*sinh(alpha*y1),x,y1,y2);
```

$$f := (x, y1, y2) \rightarrow y2$$

$$g := (x, y1, y2) \rightarrow 10.0 \sinh(10.0\, y1)$$

(5.1)

Definice parametrŢ metody

```
> a := 0:
  b := 1:
  alfa1 := 1:
  alfa2 := 1:
  beta1 := 0:
  beta2 := 0:
  gama1 := 0:
  gama2 := 1:
  eps := 0.1e-5:
  m := 10:
  h:=(b-a)/m:
  z0:=0.00035;
  Lx := evalf([seq(a+(i-1)*h, i = 1 .. m+1)]):
```

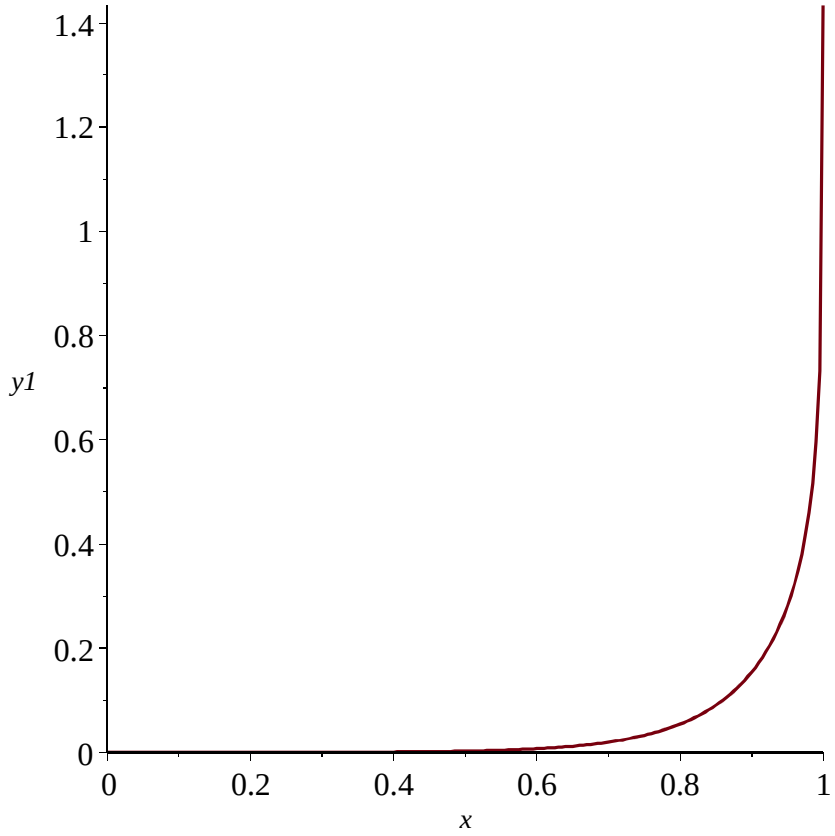
$$z0 := 0.00035$$

(5.2)

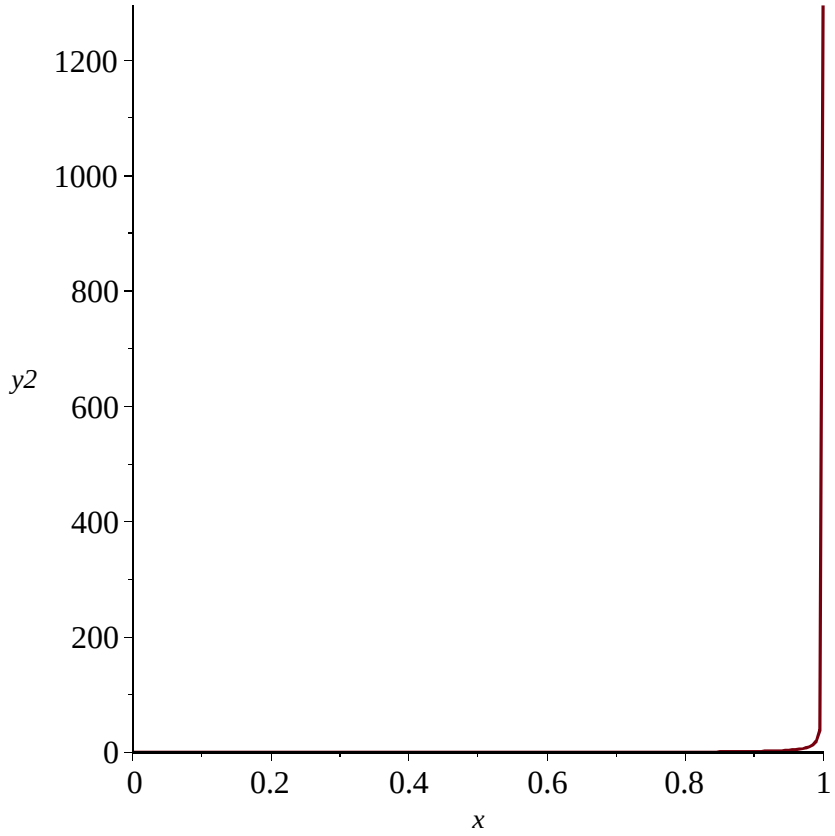
```
> v := Strelba2(f, g, a, b, alfa1,alfa2,beta1,beta2,gama1,
  gama2, eps, z0, Lx):
```

```
iterace         zn             sn
-------------------------------------
    0     0.000350000
    1     0.000363096     0.000013096
    2     0.000362694     0.000000401
```

```
> # Graf funkce y1(x)
> v[1];
```




```
> # Graf funkce y2(x)
> v[2];
```




```
> # Tabulka hodnot funkce y1(x)
> linalg[matrix](v[3]);
```

0.	0.
0.1000000000	0.0000426710375991296
0.2000000000	0.000131689676693467
0.3000000000	0.000363744588371199
0.4000000000	0.000990886299689403
0.5000000000	0.00269432439944585
0.6000000000	0.00732492757458310
0.7000000000	0.0199255582348651
0.8000000000	0.0544524484851912
0.9000000000	0.154338475958776
1.	1.76958697387172

(5.3)

```
> # Tabulka hodnot funkce y2(x)
> linalg[matrix](v[4]);
```

0.	0.000363095662115985
0.1000000000	0.000560285771045711
0.2000000000	0.00136603648329645
0.3000000000	0.00365552532641258
0.4000000000	0.00991555381562925
0.5000000000	0.0269465053651040
0.6000000000	0.0732665524232296
0.7000000000	0.199585700898439
0.8000000000	0.551276893063289
0.9000000000	1.70119441087572
1.	6960.00060569754

(5.4)

Zde jsme museli použít už pomĎrnĎ blízký nástŕel, aby metoda konvergovala.

6)

Definice parametrŢ diferenciální rovnice

```
> alpha:=20.0: # Příliš velký parametr, úlohu se nepodařilo
  vyřešit ani při z0:=0.
```

Definice pravé strany diferenciální rovnice

```
> f:=unapply(y2,x,y1,y2);
  g:=unapply(alpha*sinh(alpha*y1),x,y1,y2);
```

$$f := (x, y1, y2) \rightarrow y2$$

$$g := (x, y1, y2) \rightarrow 20.0 \sinh(20.0\, y1) \qquad \textbf{(6.1)}$$

Definice parametrŢ metody

```
> a := 0:
  b := 1:
  alfa1 := 1:
  alfa2 := 1:
  beta1 := 0:
  beta2 := 0:
  gama1 := 0:
  gama2 := 1:
  eps := 0.1e-5:
  m := 10:
  h:=(b-a)/m:
  z0:=0.0;
  Lx := evalf([seq(a+(i-1)*h, i = 1 .. m+1)]):
```

$$z0 := 0. \qquad \textbf{(6.2)}$$

```
> v := Strelba2(f, g, a, b, alfa1,alfa2,beta1,beta2,gama1,
  gama2, eps, z0, Lx):
iterace        zn           sn
-------------------------------------
    0     0.000000000
    1     0.000000082     0.000000082
Warning, cannot evaluate the solution further right of
.92388978, probably a singularity
Warning, cannot evaluate the solution further right of
.92388978, probably a singularity
```

> Zde metoda stŚelby selhala, rovnice je "stiff". Je potŚeba k Śeġení pouġít metodu sítí nebo metodu "multiple shooting".