```
Get[FileNameJoin[{NotebookDirectory[], "DESite.wl"}]]
```

Aplikační příklad 1

Apl. příklad 1: Izotermní vnitřní difuze v porézním katalyzátoru je popsána diferenciální rovnicí

$y'' + \frac{a}{x} y' = \phi^2 y^n$ s okrajovou podmínkou y'(0)=0 a y(1)=1.

Parametr a charakterizuje tvar částice katalyzátoru a=0 pro desku, a=1pro váleček, a=2 pro kuličku, n je řád reakce a ϕ Thieleho modul.

a) Vyřešte tuto rovnici metodou sítí pro n=0, a=1 a ϕ=1

b) Vyřešte tuto rovnici metodou sítí pro n=1, a=2 a ϕ=1;2;4

c) Vyřešte tuto rovnici metodou sítí pro n=1, a=0 a ϕ=1;2;4

a)

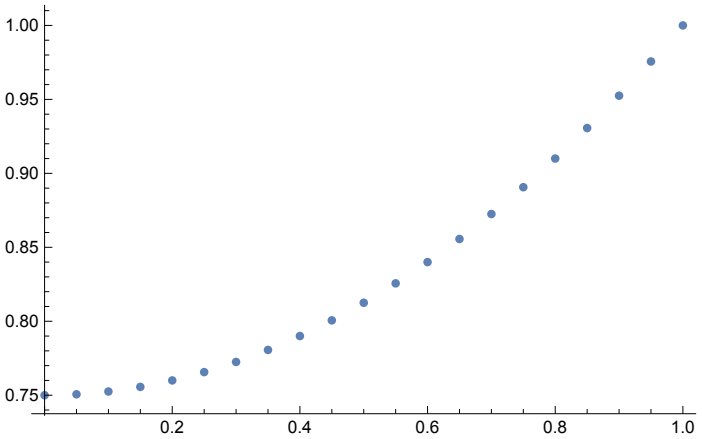
n=0, a=1 a ϕ=1

```
n = 0;
aa = 1;
ϕ = 1;

f[x_, y1_, y2_] = ϕ^2 y1^n - aa / x * y2;

a = 0.0;
b = 1.0;
α1 = 0;
α2 = 1;
β1 = 1;
β2 = 0;
γ1 = 0;
γ2 = 1;
ϵ = 0.00000001;
nn = 20;
y0 = Table[0.5, {i, 1, nn + 1}];
y0[[1]] = 1.0;
y0[[nn + 1]] = 1.0;


yres = DESite2[nn, f, a, b, α1, α2, β1, β2, γ1, γ2, ϵ, y0, 10];

iterace = 0

y={0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5,
  0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 1.}

iterace = 1     s = 0.611237

y={0.75, 0.750625, 0.7525, 0.755625, 0.76, 0.765625, 0.7725, 0.780625, 0.79, 0.800625, 0.8125,
  0.825625, 0.84, 0.855625, 0.8725, 0.890625, 0.91, 0.930625, 0.9525, 0.975625, 1.}

iterace = 2     s = 2.90343×10^-15
```

```
y={0.75, 0.750625, 0.7525, 0.755625, 0.76, 0.765625, 0.7725, 0.780625, 0.79, 0.800625, 0.8125,
  0.825625, 0.84, 0.855625, 0.8725, 0.890625, 0.91, 0.930625, 0.9525, 0.975625, 1.}
```

Graf řešení $y_1(x)$

```
ListPlot[yres]
```

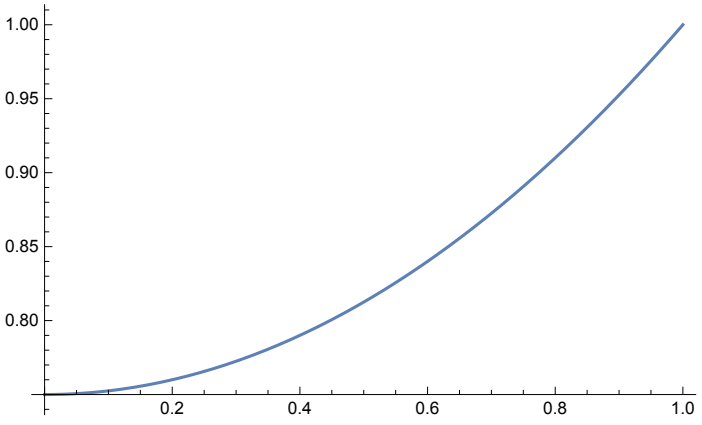


Přesné řešení

```
pres = DSolve[{y''[x] == -y'[x] / x + 1, y'[0] == 0, y[1] == 1}, y[x], x]
```

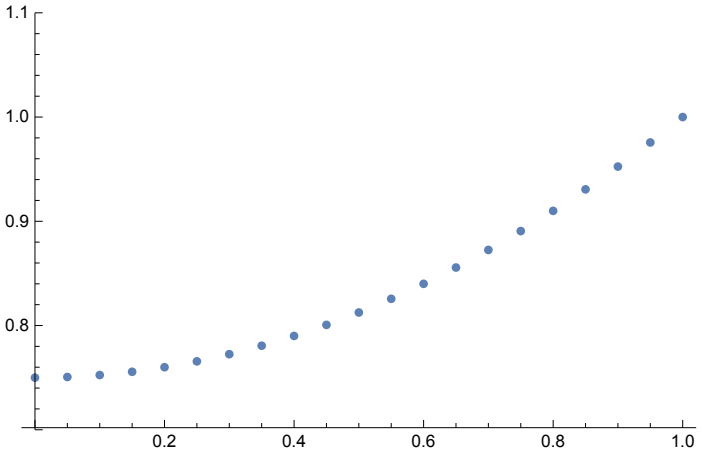
$$\left\{\left\{y[x] \to \frac{1}{4}\left(3 + x^2\right)\right\}\right\}$$

```
gp = Plot[y[x] /. pres, {x, 0, 1}]
```

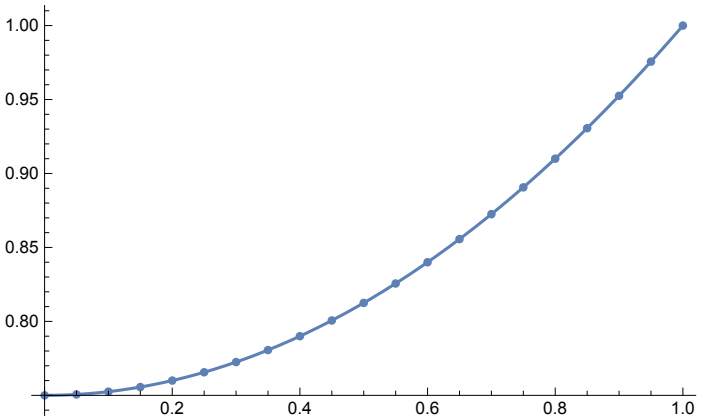


Graf řešení $y(x)$

```
g1 = ListPlot[yres, PlotRange → {0.7, 1.1}]
```




```
Show[gp, g1]
```



Tabulka řešení $y_2(x)$

```
MatrixForm[yres]
```

0.	0.75
0.05	0.750625
0.1	0.7525
0.15	0.755625
0.2	0.76
0.25	0.765625
0.3	0.7725
0.35	0.780625
0.4	0.79
0.45	0.800625
0.5	0.8125
0.55	0.825625
0.6	0.84
0.65	0.855625
0.7	0.8725
0.75	0.890625
0.8	0.91
0.85	0.930625
0.9	0.9525
0.95	0.975625
1.	1.

b)

n=1, a=2 a ϕ=1

```
n = 1;
aa = 2;
ϕ = 1;
```

```
f[x_, y_, dy_] = ϕ^2 y^n - aa / x * dy
```

$$-\frac{2\,dy}{x} + y$$

```
a = 0.0;
b = 1.0;
α1 = 0;
α2 = 1;
β1 = 1;
β2 = 0;
γ1 = 0;
γ2 = 1;
ϵ = 0.000001;
nn = 20;
y0 = Table[0.8, {i, 1, nn + 1}];
y0[[1]] = 1.0;
y0[[nn + 1]] = 1.0;
```

```
yres = DESite2[nn, f, a, b, α1, α2 , β1, β2 , γ1 , γ2 , ϵ, y0, 10];
```

```
iterace =  0
```

```
y={0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8,
  0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 1.}
```

```
iterace = 1     s = 0.126082
```

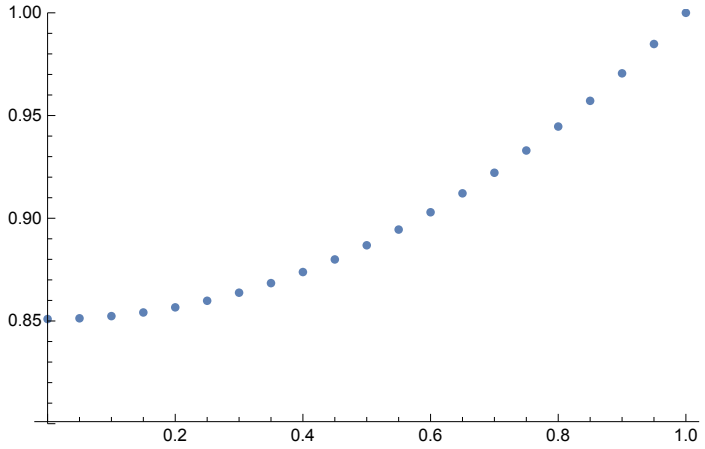
```
y={0.850946, 0.8513, 0.852365, 0.85414, 0.856629, 0.859836,
  0.863765, 0.868422, 0.873815, 0.879951, 0.88684, 0.894492, 0.902919,
  0.912132, 0.922147, 0.932978, 0.944643, 0.957157, 0.970541, 0.984815, 1.}
```

```
iterace = 2     s = 4.27998×10^-16
```

```
y={0.850946, 0.8513, 0.852365, 0.85414, 0.856629, 0.859836,
  0.863765, 0.868422, 0.873815, 0.879951, 0.88684, 0.894492, 0.902919,
  0.912132, 0.922147, 0.932978, 0.944643, 0.957157, 0.970541, 0.984815, 1.}
```

Graf řešení *y*(x)

```
g1 = ListPlot[yres, PlotRange → {0.8, 1}]
```



Tabulka řešení *y*(x)

```
MatrixForm[yres]
```

0.	0.850946
0.05	0.8513
0.1	0.852365
0.15	0.85414
0.2	0.856629
0.25	0.859836
0.3	0.863765
0.35	0.868422
0.4	0.873815
0.45	0.879951
0.5	0.88684
0.55	0.894492
0.6	0.902919
0.65	0.912132
0.7	0.922147
0.75	0.932978
0.8	0.944643
0.85	0.957157
0.9	0.970541
0.95	0.984815
1.	1.

n=1, a=2 a ϕ=2

```
n = 1;
aa = 2;
ϕ = 2;
```

```
f[x_, y_, dy_] = ϕ^2 y^n - aa / x * dy
```

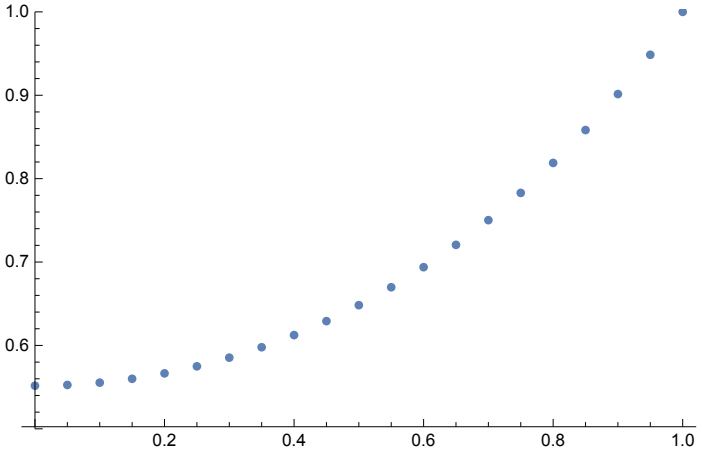
$$-\frac{2\,dy}{x}+4\,y$$

```
a = 0.0;
b = 1.0;
α1 = 0;
α2 = 1;
β1 = 1;
β2 = 0;
γ1 = 0;
γ2 = 1;
ϵ = 0.000001;
nn = 20;
y0 = Table[0.8, {i, 1, nn + 1}];
y0[[1]] = 1.0;
y0[[nn + 1]] = 1.0;

yres = DESite2[nn, f, a, b, α1, α2 , β1, β2 , γ1 , γ2 , ϵ, y0, 10];
```

```
iterace =  0

y={0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8,
  0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 1.}

iterace = 1     s = 0.207337

y={0.551686, 0.552607, 0.55537, 0.559993, 0.566505, 0.574944,
  0.585361, 0.59782, 0.612394, 0.629174, 0.64826, 0.669769, 0.693833,
  0.720599, 0.750233, 0.782918, 0.818857, 0.858274, 0.901418, 0.948561, 1.}

iterace = 2     s = 1.69367×10^-15

y={0.551686, 0.552607, 0.55537, 0.559993, 0.566505, 0.574944,
  0.585361, 0.59782, 0.612394, 0.629174, 0.64826, 0.669769, 0.693833,
  0.720599, 0.750233, 0.782918, 0.818857, 0.858274, 0.901418, 0.948561, 1.}
```

Graf řešení *y*(x)

```
g1 = ListPlot[yres, PlotRange → {0.5, 1}]
```



Tabulka řešení *y*(x)

```
MatrixForm[yres]
```

0.	0.551686
0.05	0.552607
0.1	0.55537
0.15	0.559993
0.2	0.566505
0.25	0.574944
0.3	0.585361
0.35	0.59782
0.4	0.612394
0.45	0.629174
0.5	0.64826
0.55	0.669769
0.6	0.693833
0.65	0.720599
0.7	0.750233
0.75	0.782918
0.8	0.818857
0.85	0.858274
0.9	0.901418
0.95	0.948561
1.	1.

n=1, a=2 a ϕ=4

```
n = 1;
aa = 2;
ϕ = 4;
```

```
f[x_, y_, dy_] = ϕ^2 y^n - aa / x * dy
```

$$-\frac{2\,dy}{x}+16\,y$$

```
a = 0.0;
b = 1.0;
α1 = 0;
α2 = 1;
β1 = 1;
β2 = 0;
γ1 = 0;
γ2 = 1;
ϵ = 0.000001;
nn = 20;
y0 = Table[0.8, {i, 1, nn + 1}];
y0[[1]] = 1.0;
y0[[nn + 1]] = 1.0;
```

```
yres = DESite2[nn, f, a, b, α1, α2 , β1, β2 , γ1 , γ2 , ϵ, y0, 10];
```

```
iterace =  0
```

```
y={0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8,
  0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 1.}
```

```
iterace = 1     s = 0.609405
```

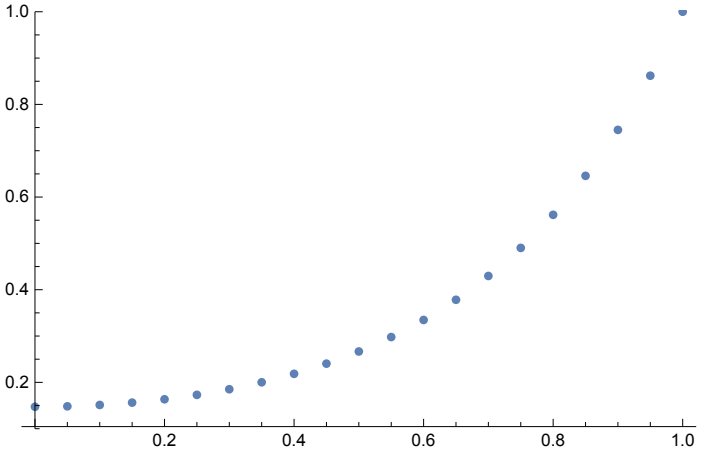
```
y={0.147298, 0.148287, 0.151253, 0.156275, 0.163474, 0.173025,
  0.185159, 0.200175, 0.218443, 0.240418, 0.266654, 0.297816, 0.334704,
  0.378275, 0.429672, 0.490257, 0.561654, 0.645795, 0.744985, 0.861964, 1.}
```

```
iterace = 2     s = 1.09586×10^-15
```

```
y={0.147298, 0.148287, 0.151253, 0.156275, 0.163474, 0.173025,
  0.185159, 0.200175, 0.218443, 0.240418, 0.266654, 0.297816, 0.334704,
  0.378275, 0.429672, 0.490257, 0.561654, 0.645795, 0.744985, 0.861964, 1.}
```

Graf řešení *y*(x)

```
g1 = ListPlot[yres, PlotRange → {0.1, 1}]
```



Tabulka řešení *y*(x)

```
MatrixForm[yres]
```

0.	0.147298
0.05	0.148287
0.1	0.151253
0.15	0.156275
0.2	0.163474
0.25	0.173025
0.3	0.185159
0.35	0.200175
0.4	0.218443
0.45	0.240418
0.5	0.266654
0.55	0.297816
0.6	0.334704
0.65	0.378275
0.7	0.429672
0.75	0.490257
0.8	0.561654
0.85	0.645795
0.9	0.744985
0.95	0.861964
1.	1.

c)

n=1, a=0 a ϕ=1

```
n = 1;
aa = 0;
ϕ = 1;
```

```
f[x_, y_, dy_] = ϕ^2 y^n - aa / x * dy
```

y

```
a = 0.0;
b = 1.0;
α1 = 0;
α2 = 1;
β1 = 1;
β2 = 0;
γ1 = 0;
γ2 = 1;
ϵ = 0.000001;
nn = 20;
y0 = Table[0.8, {i, 1, nn + 1}];
y0[[1]] = 1.0;
y0[[nn + 1]] = 1.0;
```

```
yres = DESite2[nn, f, a, b, α1, α2 , β1, β2 , γ1 , γ2 , ϵ, y0, 10];
```

```
iterace =  0
```

```
y={0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8,
  0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 1.}
```

```
iterace = 1     s = 0.131157
```

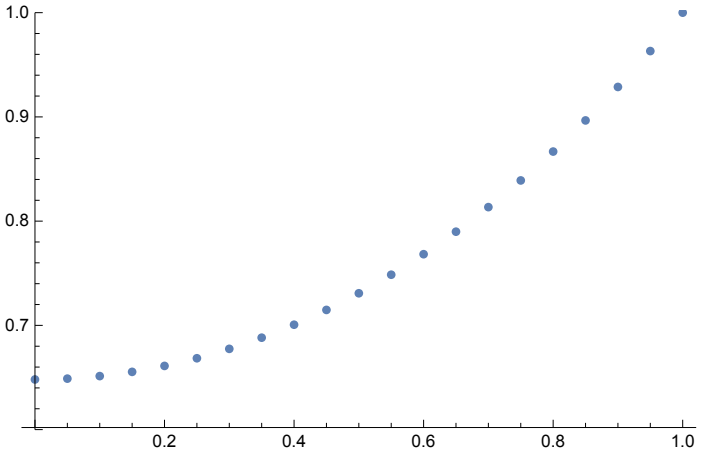
```
y={0.64809, 0.648901, 0.651335, 0.655396, 0.661097, 0.66845,
  0.677474, 0.688191, 0.70063, 0.714819, 0.730796, 0.7486, 0.768276,
  0.789872, 0.813442, 0.839047, 0.866749, 0.896617, 0.928728, 0.96316, 1.}
```

```
iterace = 2     s = 5.89213×10^-16
```

```
y={0.64809, 0.648901, 0.651335, 0.655396, 0.661097, 0.66845,
  0.677474, 0.688191, 0.70063, 0.714819, 0.730796, 0.7486, 0.768276,
  0.789872, 0.813442, 0.839047, 0.866749, 0.896617, 0.928728, 0.96316, 1.}
```

Graf řešení *y*(x)

```
g1 = ListPlot[yres, PlotRange → {0.6, 1}]
```



Tabulka řešení *y*(x)

```
MatrixForm[yres]
```

0.	0.64809
0.05	0.648901
0.1	0.651335
0.15	0.655396
0.2	0.661097
0.25	0.66845
0.3	0.677474
0.35	0.688191
0.4	0.70063
0.45	0.714819
0.5	0.730796
0.55	0.7486
0.6	0.768276
0.65	0.789872
0.7	0.813442
0.75	0.839047
0.8	0.866749
0.85	0.896617
0.9	0.928728
0.95	0.96316
1.	1.

n=1, a=0 a ϕ=2

```
n = 1;
aa = 0;
ϕ = 2;
```

```
f[x_, y_, dy_] = ϕ^2 y^n - aa / x * dy
```

4 y

```
a = 0.0;
b = 1.0;
α1 = 0;
α2 = 1;
β1 = 1;
β2 = 0;
γ1 = 0;
γ2 = 1;
ϵ = 0.000001;
nn = 20;
y0 = Table[0.8, {i, 1, nn + 1}];
y0[[1]] = 1.0;
y0[[nn + 1]] = 1.0;
```

```
yres = DESite2[nn, f, a, b, α1, α2 , β1, β2 , γ1 , γ2 , ϵ, y0, 10];
```

```
iterace =  0
```

```
y={0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8,
  0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 1.}
```

```
iterace = 1     s = 0.468255
```

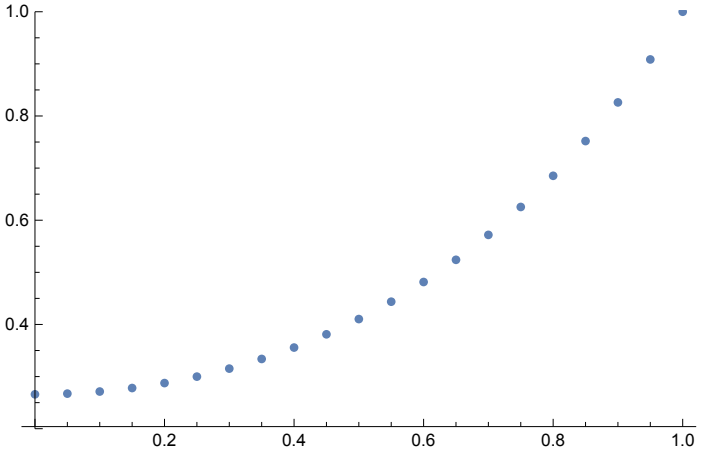
```
y={0.265951, 0.267288, 0.271297, 0.278019, 0.287522, 0.299899,
  0.315276, 0.333806, 0.355673, 0.381097, 0.410333, 0.443671, 0.481446,
  0.524036, 0.571866, 0.625415, 0.685218, 0.751873, 0.826047, 0.908481, 1.}
```

```
iterace = 2     s = 1.01445×10^-14
```

```
y={0.265951, 0.267288, 0.271297, 0.278019, 0.287522, 0.299899,
  0.315276, 0.333806, 0.355673, 0.381097, 0.410333, 0.443671, 0.481446,
  0.524036, 0.571866, 0.625415, 0.685218, 0.751873, 0.826047, 0.908481, 1.}
```

Graf řešení *y*(x)

```
g1 = ListPlot[yres, PlotRange → {0.2, 1}]
```



Tabulka řešení *y*(x)

```
MatrixForm[yres]
```

0.	0.265951
0.05	0.267288
0.1	0.271297
0.15	0.278019
0.2	0.287522
0.25	0.299899
0.3	0.315276
0.35	0.333806
0.4	0.355673
0.45	0.381097
0.5	0.410333
0.55	0.443671
0.6	0.481446
0.65	0.524036
0.7	0.571866
0.75	0.625415
0.8	0.685218
0.85	0.751873
0.9	0.826047
0.95	0.908481
1.	1.

n=1, a=0 a ϕ=4

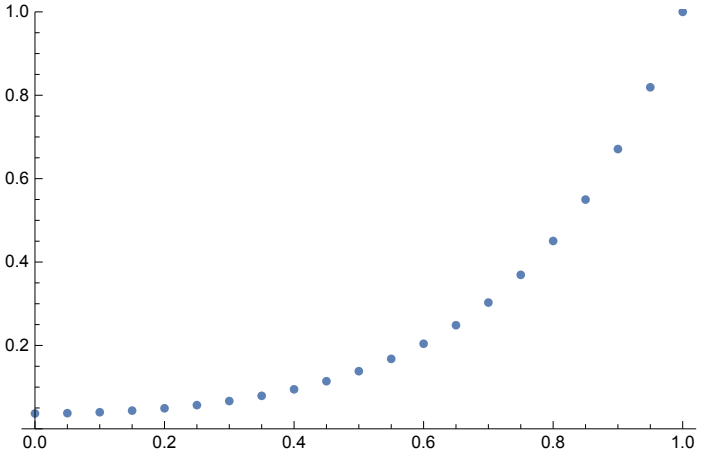
```
n = 1;
aa = 0;
ϕ = 4;

f[x_, y_, dy_] = ϕ^2 y^n - aa / x * dy
```

16 y

```
a = 0.0;
b = 1.0;
α1 = 0;
α2 = 1;
β1 = 1;
β2 = 0;
γ1 = 0;
γ2 = 1;
ϵ = 0.000001;
nn = 20;
y0 = Table[0.8, {i, 1, nn + 1}];
y0[[1]] = 1.0;
y0[[nn + 1]] = 1.0;

yres = DESite2[nn, f, a, b, α1, α2 , β1, β2 , γ1 , γ2 , ϵ, y0, 10];

iterace =  0

y={0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8,
  0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 1.}

iterace = 1     s = 0.741451

y={0.036788, 0.0375388, 0.0397911, 0.0436351, 0.0492245, 0.0567828,
  0.0666125, 0.0791067, 0.0947651, 0.114214, 0.138232, 0.167779, 0.204037,
  0.248456, 0.302814, 0.369284, 0.450526, 0.549788, 0.671043, 0.819139, 1.}

iterace = 2     s = 1.30413×10^-15

y={0.036788, 0.0375388, 0.0397911, 0.0436351, 0.0492245, 0.0567828,
  0.0666125, 0.0791067, 0.0947651, 0.114214, 0.138232, 0.167779, 0.204037,
  0.248456, 0.302814, 0.369284, 0.450526, 0.549788, 0.671043, 0.819139, 1.}
```

Graf řešení *y*(x)

```
g1 = ListPlot[yres, PlotRange → {0.0, 1}]
```



Tabulka řešení *y*(x)

```
MatrixForm[yres]
```

0.	0.036788
0.05	0.0375388
0.1	0.0397911
0.15	0.0436351
0.2	0.0492245
0.25	0.0567828
0.3	0.0666125
0.35	0.0791067
0.4	0.0947651
0.45	0.114214
0.5	0.138232
0.55	0.167779
0.6	0.204037
0.65	0.248456
0.7	0.302814
0.75	0.369284
0.8	0.450526
0.85	0.549788
0.9	0.671043
0.95	0.819139
1.	1.