In[351]:=
```
Get[FileNameJoin[{NotebookDirectory[], "DeSite.wl"}]]
```

Aplikační příklad 5

Apl. příklad 5: V inženýrských aplikacích se zřídka setkáváme s úlohami, které nemají řešení. Tento příklad uvádí jeden z takových případů. Průběh exotermní reakce u explozivních materiálů lze popsat rovnicí

$\frac{\partial^2 \theta}{\partial x^2} = -\delta \ e^{(\theta)}$,

s okrajovými podmínkami

$\frac{\partial \theta}{\partial x}(0) = 0, \ \theta(1) = 0.$

Pro $\delta<\delta^*$ má tato rovnice pro dané okrajové podmínky řešení, pro $\delta>\delta^*$ toto řešení neexistuje (v takovém případě dochází k výbuchu). Ukažte, že hodnota tohoto kritického parametru δ^* je $\delta^*=0{,}8785$. Nám vyšla hodnota $\delta^*=0{,}93719$. Přesná hodnota je $\delta^*=0{,}93727$, hodnata závisí na přesnosti počítače i metody.

Rovnici převedeme substitucí δ x = z na rovnici:

$\frac{\partial^2 \theta}{\partial z^2} = - e^{(\theta)}$,

s okrajovými podmínkami

$\frac{\partial \theta}{\partial z}(0) = 0, \ \theta(\delta) = 0.$

Definice parametrů diferenciální rovnice

In[352]:=
```
δ0 = 0.93719
```

Out[352]=
```
0.93719
```

Definice pravé strany diferenciální rovnice

In[353]:=
```
f[z_, θ1_, θ2_] = - Exp[θ1];
```

Parametry programu

In[354]:=
```
a = 0.0;
b = δ0;
α1 = 0;
α2 = 1;
β1 = 1;
β2 = 0;
γ1 = 0;
γ2 = 0;
ϵ = 0.00001;
n = 20;
θ0 = Table[0.5, {i, 1, n + 1}];
θ0[[1]] = 1.0;
θ0[[n + 1]] = 1.0;
```

In[367]:=
```
For[i = 0, i < 2, i++, {
  b = δ0  + i * 0.00001;
  Print["i=", i, "    δ=", b];
  Θres = DESite2[n, f, a, b, α1, α2 , β1, β2 , ϒ1 , ϒ2 , ϵ, Θ0, 15];
  Print["Θ(δ)=", Θres[[n]][[2]]];
 }]
```

```
i=0    δ=0.93719

iterace =  0

y={0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5,
  0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.}

iterace = 1     s = 0.557441

y={0.892406, 0.88989, 0.882343, 0.86979, 0.852279, 0.829872,
  0.802651, 0.770714, 0.734176, 0.69317, 0.647845, 0.598364, 0.544907,
  0.487667, 0.426851, 0.36268, 0.295386, 0.225212, 0.152412, 0.0772511, 0.}

iterace = 2     s = 0.166378

y={1.04626, 1.04318, 1.03393, 1.01857, 0.997192, 0.969917,
  0.936902, 0.89833, 0.854405, 0.805356, 0.751421, 0.692855, 0.629917,
  0.562871, 0.491979, 0.417504, 0.3397, 0.258815, 0.175086, 0.0887429, 0.}

iterace = 3     s = 0.0666947

y={1.118, 1.11466, 1.10464, 1.08802, 1.06489, 1.0354,
  0.999742, 0.958129, 0.910802, 0.858023, 0.800072, 0.73724, 0.669823,
  0.598118, 0.522423, 0.443027, 0.360212, 0.27425, 0.185399, 0.0939055, 0.}

iterace = 4     s = 0.030242

y={1.1527, 1.14924, 1.13885, 1.12161, 1.09763, 1.06707,
  1.03014, 0.987051, 0.938076, 0.883493, 0.823599, 0.758703, 0.689119,
  0.615162, 0.537143, 0.455367, 0.370129, 0.281712, 0.190385, 0.0964015, 0.}

iterace = 5     s = 0.0143709

y={1.16969, 1.16617, 1.1556, 1.13805, 1.11366, 1.08258,
  1.04502, 1.00121, 0.951429, 0.895962, 0.835117, 0.76921, 0.698565,
  0.623505, 0.544348, 0.461408, 0.374984, 0.285365, 0.192826, 0.0976234, 0.}

iterace = 6     s = 0.00687449

y={1.17793, 1.17438, 1.16372, 1.14603, 1.12144, 1.0901,
  1.05224, 1.00808, 0.957907, 0.902012, 0.840705, 0.774308, 0.703148,
  0.627553, 0.547845, 0.464339, 0.37734, 0.287138, 0.19401, 0.0982162, 0.}

iterace = 7     s = 0.0031157

y={1.1817, 1.17813, 1.16743, 1.14968, 1.12499, 1.09354,
  1.05553, 1.01122, 0.960864, 0.904773, 0.843255, 0.776635, 0.70524,
  0.6294, 0.54944, 0.465676, 0.378415, 0.287947, 0.19455, 0.0984868, 0.}

iterace = 8     s = 0.00112056

y={1.18305, 1.17948, 1.16877, 1.15099, 1.12627, 1.09477,
  1.05672, 1.01235, 0.961931, 0.905769, 0.844175, 0.777474, 0.705995,
  0.630067, 0.550016, 0.466159, 0.378802, 0.288239, 0.194745, 0.0985844, 0.}

iterace = 9     s = 0.000197676

y={1.18329, 1.17972, 1.16901, 1.15122, 1.12649, 1.09499,
  1.05693, 1.01255, 0.962119, 0.905945, 0.844338, 0.777622, 0.706128,
  0.630185, 0.550118, 0.466244, 0.378871, 0.28829, 0.19478, 0.0986016, 0.}
```

iterace = 10 s = 6.58056×10^{-6}

y={1.1833, 1.17973, 1.16901, 1.15123, 1.1265, 1.095,
1.05694, 1.01255, 0.962125, 0.905951, 0.844343, 0.777627, 0.706132,
0.630188, 0.550121, 0.466247, 0.378873, 0.288292, 0.194781, 0.0986022, 0.}

$\Theta(\delta)$=0.0986022

i=1 δ=0.9372

iterace = 0

y={0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5,
0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.}

iterace = 1 s = 0.557489

y={0.892453, 0.889937, 0.882389, 0.869836, 0.852324, 0.829916,
0.802693, 0.770754, 0.734214, 0.693206, 0.647878, 0.598395, 0.544935,
0.487692, 0.426873, 0.362698, 0.2954, 0.225223, 0.15242, 0.0772549, 0.}

iterace = 2 s = 0.166451

y={1.04639, 1.0433, 1.03405, 1.01869, 0.997308, 0.970029,
0.93701, 0.898432, 0.854503, 0.805447, 0.751506, 0.692933, 0.629987,
0.562933, 0.492033, 0.417549, 0.339736, 0.258842, 0.175105, 0.0887521, 0.}

iterace = 3 s = 0.0668245

y={1.11827, 1.11493, 1.10491, 1.08828, 1.06514, 1.03565,
0.99998, 0.958355, 0.911015, 0.858223, 0.800257, 0.737409, 0.669975,
0.598253, 0.522539, 0.443124, 0.36029, 0.274309, 0.185439, 0.0939254, 0.}

iterace = 4 s = 0.0304917

y={1.15327, 1.1498, 1.13941, 1.12216, 1.09816, 1.06759,
1.03063, 0.987523, 0.938522, 0.883909, 0.823984, 0.759054, 0.689435,
0.615441, 0.537384, 0.45557, 0.370292, 0.281835, 0.190467, 0.0964426, 0.}

iterace = 5 s = 0.0148671

y={1.17085, 1.16732, 1.15674, 1.13918, 1.11475, 1.08364,
1.04603, 1.00218, 0.952342, 0.896815, 0.835905, 0.769929, 0.699212,
0.624076, 0.544842, 0.461822, 0.375317, 0.285616, 0.192993, 0.0977072, 0.}

iterace = 6 s = 0.00790342

y={1.18034, 1.17678, 1.16609, 1.14836, 1.12371, 1.0923,
1.05434, 1.01008, 0.959797, 0.903777, 0.842336, 0.775796, 0.704486,
0.628735, 0.548866, 0.465195, 0.378028, 0.287656, 0.194356, 0.0983895, 0.}

iterace = 7 s = 0.00565276

y={1.18718, 1.18359, 1.17283, 1.15498, 1.13016, 1.09854,
1.06033, 1.01578, 0.965172, 0.908796, 0.846972, 0.780025, 0.708289,
0.632093, 0.551766, 0.467627, 0.379982, 0.289126, 0.195339, 0.0988813, 0.}

iterace = 8 s = 0.0607314

y={1.11329, 1.10996, 1.1, 1.08345, 1.06044, 1.0311,
0.995617, 0.954204, 0.907101, 0.854568, 0.796882, 0.73433, 0.667207,
0.595809, 0.520428, 0.441355, 0.358868, 0.273239, 0.184724, 0.0935675, 0.}

iterace = 9 s = 0.0328302

y={1.15081, 1.14735, 1.13698, 1.11977, 1.09584, 1.06534,
1.02848, 0.985472, 0.936587, 0.882103, 0.822315, 0.757532, 0.688067,
0.614232, 0.53634, 0.454694, 0.369589, 0.281306, 0.190113, 0.0962656, 0.}

iterace = 10 s = 0.0159115

```
y={1.16959, 1.16606, 1.15549, 1.13795, 1.11356, 1.08248,
  1.04492, 1.00112, 0.951347, 0.895886, 0.835047, 0.769147, 0.698508,
  0.623455, 0.544306, 0.461372, 0.374955, 0.285344, 0.192811, 0.0976162, 0.}

iterace = 11     s = 0.0083532

y={1.1796, 1.17604, 1.16537, 1.14765, 1.12301, 1.09163,
  1.0537, 1.00947, 0.959219, 0.903237, 0.841837, 0.775341, 0.704077,
  0.628374, 0.548554, 0.464933, 0.377817, 0.287498, 0.19425, 0.0983366, 0.}

iterace = 12     s = 0.00566238

y={1.18645, 1.18287, 1.17212, 1.15428, 1.12947, 1.09787,
  1.05969, 1.01518, 0.9646, 0.908262, 0.846478, 0.779575, 0.707884,
  0.631736, 0.551457, 0.467368, 0.379774, 0.28897, 0.195234, 0.0988289, 0.}

iterace = 13     s = 0.0464745

y={1.24296, 1.23917, 1.22782, 1.20898, 1.18279, 1.14945,
  1.10919, 1.06227, 1.00901, 0.949735, 0.884786, 0.814522, 0.739302,
  0.659486, 0.575424, 0.48746, 0.395921, 0.30112, 0.203352, 0.102893, 0.}

iterace = 14     s = 0.0221757

y={1.21474, 1.21106, 1.2, 1.18166, 1.15617, 1.1237,
  1.08447, 1.03876, 0.986836, 0.929026, 0.865658, 0.797072, 0.723614,
  0.645629, 0.563457, 0.477427, 0.387858, 0.295053, 0.199298, 0.100864, 0.}

iterace = 15     s = 0.0118189

y={1.20004, 1.1964, 1.18551, 1.16743, 1.14229, 1.11028,
  1.07159, 1.0265, 0.975279, 0.918234, 0.855689, 0.787978, 0.715438,
  0.638408, 0.55722, 0.472199, 0.383656, 0.291891, 0.197186, 0.0998061, 0.}

Vyčerpán maximální počet iterací, počet iterací = 15

Θ(δ) =0.0998061
```

Pro hodnotu parametru δ = 0.9372 již nedostaneme řešení.

Výpočet přesné hodnoty δ^*

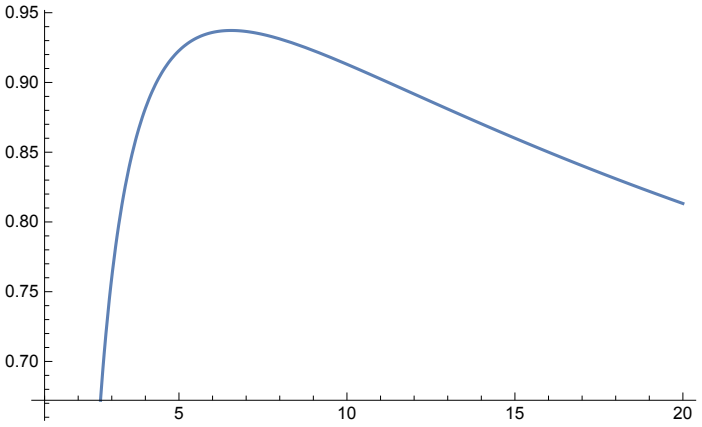
Přesné řešení je $\theta(x) = \ln\left(\frac{1}{2}\, a\, (1 - \tanh^2\left(\frac{1}{2} a\, x^2\right))\right)$, konstantu a získáme z podmínky $\theta(\delta)=0$

Výpočet kritického δ^*

graf závislosti $\delta(a)$, kde $a = 2\, e^{\theta(0)}$

In[368]:=
```
Plot[2 ArcTanh[Sqrt[1 - 2 / a]] / Sqrt[a], {a, 1, 20}]
```

Out[368]=



In[369]:=
```
a = .
```

In[370]:=
```
rov = D[2 ArcTanh[Sqrt[1 - 2 / a]] / Sqrt[a], a] == 0
```

Out[370]=
$$\frac{1}{\sqrt{1-\frac{2}{a}}\,a^{3/2}}-\frac{\text{ArcTanh}\left[\sqrt{1-\frac{2}{a}}\right]}{a^{3/2}}==0$$

In[371]:=
```
a1 = FindRoot[rov, {a, 6}]
```

Out[371]=
```
{a → 6.55344}
```

Kritická hodnota parametru δ

In[372]:=
```
δ = 2 ArcTanh[Sqrt[1 - 2 / a]] / Sqrt[a] /. a1
```

Out[372]=
```
0.937261
```