

```
In[1]:= Get[FileNameJoin[{NotebookDirectory[], "DeStrelba.wl"}]]
```

Aplikační příklad 6

Apl. příklad 6: Jednoduchý model chování sloupce plasmy stlačované zářením odvodil Troesch ve tvaru nelineární okrajové úlohy

$$\frac{\partial^2 y}{\partial x^2} = \alpha \sinh(\alpha y) ,$$

$$y(0) = 0, \quad y(1) = 1.$$

Použijte parametry $\alpha = 0,8; 1; 2; 5; 10; 20$.

$\alpha = 0.8$

Definice parametrů diferenciální rovnice

```
In[2]:=  $\alpha = 0.8;$ 
```

Definice pravé strany diferenciální rovnice

```
In[3]:= f[x_, y1_, y2_] = y2;  
g[x_, y1_, y2_] =  $\alpha \sinh[\alpha y1];$ 
```

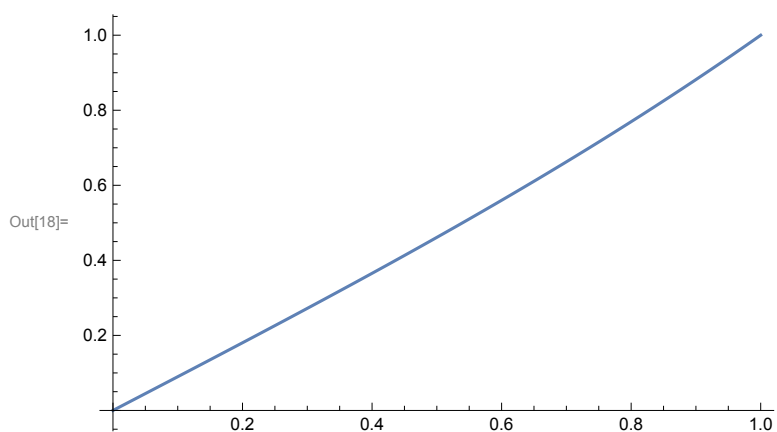
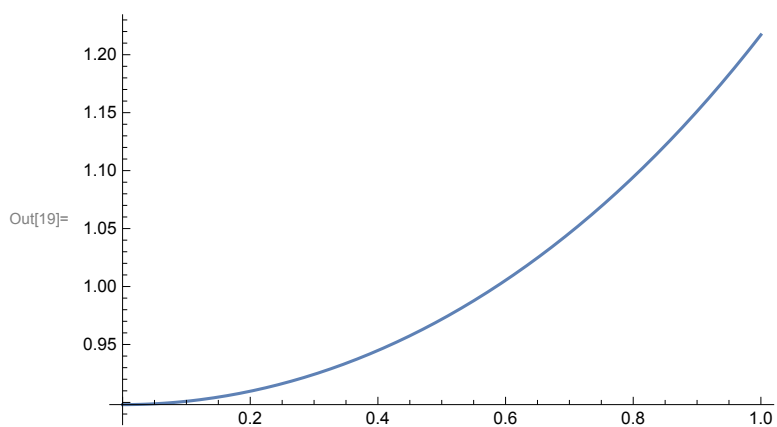
Parametry programu Strelba2

```
In[5]:= a = 0.0;  
b = 1.0;  
 $\alpha 1 = 1;$   
 $\alpha 2 = 1;$   
 $\beta 1 = 0;$   
 $\beta 2 = 0;$   
 $\gamma 1 = 0;$   
 $\gamma 2 = 1;$   
 $\epsilon = 0.000001;$   
 $z0 = 1.0;$   
m = 10;  
Lx = Table[N[a + i (b - a) / m], {i, 0, m}];
```

```
In[17]:= v = Strelba2[f, g, a, b,  $\alpha 1$ ,  $\alpha 2$ ,  $\beta 1$ ,  $\beta 2$ ,  $\gamma 1$ ,  $\gamma 2$ ,  $\epsilon$ , z0, Lx];
```

i	z	s
0	1.	
1	0.898231	0.101769
2	0.898115	0.000115411
3	0.898115	1.39111×10^{-10}

Graf řešení $y_1(x)$

In[18]:= **v[[1]]**Graf řešení $y_2(x)$ In[19]:= **v[[2]]**Tabulka řešení $y_1(x)$ In[20]:= **MatrixForm[v[[3]]]**

Out[20]//MatrixForm=

$$\begin{pmatrix} 0. & 0. \\ 0.1 & 0.0899074 \\ 0.2 & 0.180391 \\ 0.3 & 0.272035 \\ 0.4 & 0.365435 \\ 0.5 & 0.461209 \\ 0.6 & 0.560007 \\ 0.7 & 0.662513 \\ 0.8 & 0.769466 \\ 0.9 & 0.881668 \\ 1. & 1. \end{pmatrix}$$
Tabulka řešení $y_2(x)$

In[21]:= **MatrixForm**[v[[4]]]

Out[21]//MatrixForm=

$$\begin{pmatrix} 0. & 0.898115 \\ 0.1 & 0.900992 \\ 0.2 & 0.909656 \\ 0.3 & 0.924208 \\ 0.4 & 0.944822 \\ 0.5 & 0.971751 \\ 0.6 & 1.00534 \\ 0.7 & 1.04602 \\ 0.8 & 1.09438 \\ 0.9 & 1.15112 \\ 1. & 1.21716 \end{pmatrix}$$

$\alpha = 1.0$

Definice parametrů diferenciální rovnice

In[22]:= **$\alpha = 1.0$** ;

Definice pravé strany diferenciální rovnice

In[23]:= **f[x_, y1_, y2_] = y2;**
g[x_, y1_, y2_] = α Sinh[α y1];

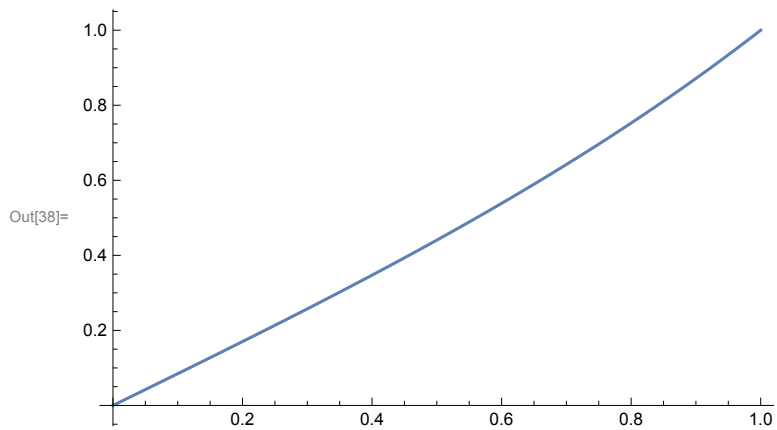
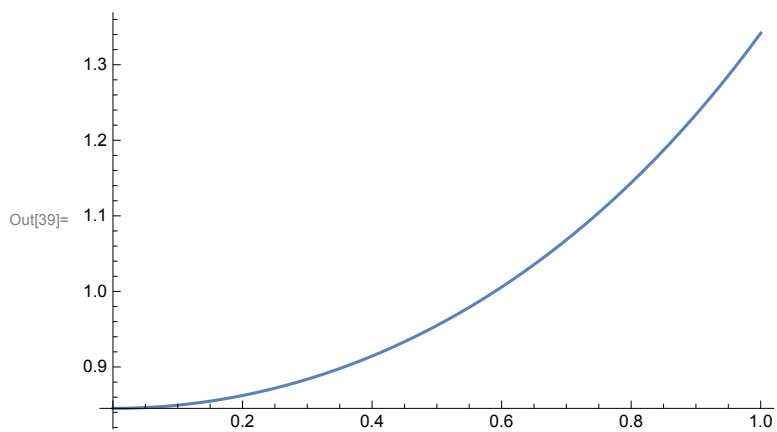
Parametry programu Strelba2

In[25]:= **a = 0.0;**
b = 1.0;
 $\alpha 1 = 1$;
 $\alpha 2 = 1$;
 $\beta 1 = 0$;
 $\beta 2 = 0$;
 $\gamma 1 = 0$;
 $\gamma 2 = 1$;
 $\epsilon = 0.000001$;
z0 = 1.0;
m = 10;
Lx = Table[N[a + i (b - a) / m], {i, 0, m}];

In[37]:= **v = Strelba2[f, g, a, b, $\alpha 1$, $\alpha 2$, $\beta 1$, $\beta 2$, $\gamma 1$, $\gamma 2$, ϵ , z0, Lx];**

i	z	s
0	1.	
1	0.845888	0.154112
2	0.845203	0.000684959
3	0.845203	1.17771×10^{-8}

Graf řešení $y_1(x)$

In[38]:= **v[[1]]**Graf řešení $y_2(x)$ In[39]:= **v[[2]]**Tabulka řešení $y_1(x)$ In[40]:= **MatrixForm[v[[3]]]**

Out[40]//MatrixForm=

$$\begin{pmatrix} 0. & 0. \\ 0.1 & 0.0846612 \\ 0.2 & 0.170171 \\ 0.3 & 0.257394 \\ 0.4 & 0.347223 \\ 0.5 & 0.4406 \\ 0.6 & 0.538534 \\ 0.7 & 0.642129 \\ 0.8 & 0.752608 \\ 0.9 & 0.871363 \\ 1. & 1. \end{pmatrix}$$
Tabulka řešení $y_2(x)$

```
In[41]:= MatrixForm[v[[4]]]
```

```
Out[41]//MatrixForm=
```

$$\begin{pmatrix} 0. & 0.845203 \\ 0.1 & 0.849435 \\ 0.2 & 0.862204 \\ 0.3 & 0.883734 \\ 0.4 & 0.914411 \\ 0.5 & 0.954807 \\ 0.6 & 1.00572 \\ 0.7 & 1.0682 \\ 0.8 & 1.14369 \\ 0.9 & 1.23406 \\ 1. & 1.34184 \end{pmatrix}$$

$\alpha = 2.0$

Definice parametrů diferenciální rovnice

```
In[42]:=  $\alpha = 2.0;$ 
```

Definice pravé strany diferenciální rovnice

```
In[43]:= f[x_, y1_, y2_] = y2;  
g[x_, y1_, y2_] =  $\alpha \sinh[\alpha y1];$ 
```

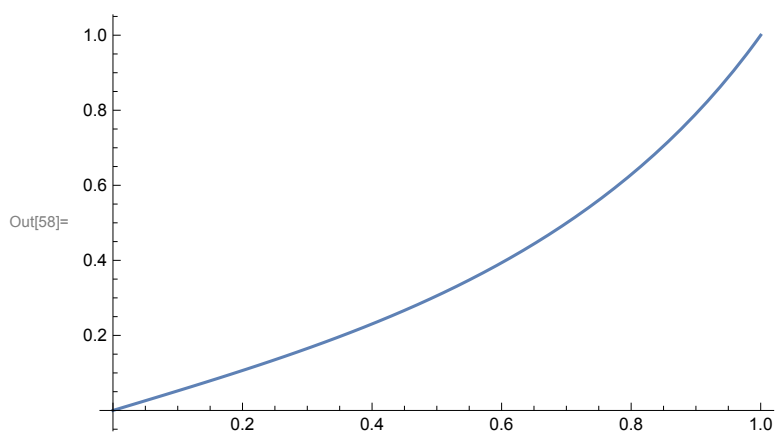
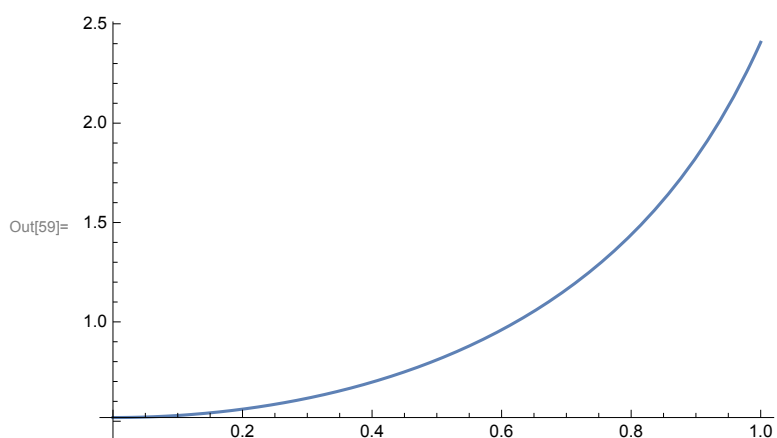
Parametry programu Strelba2

```
In[45]:= a = 0.0;  
b = 1.0;  
 $\alpha 1 = 1;$   
 $\alpha 2 = 1;$   
 $\beta 1 = 0;$   
 $\beta 2 = 0;$   
 $\gamma 1 = 0;$   
 $\gamma 2 = 1;$   
 $\epsilon = 0.000001;$   
 $z0 = 1.0;$   
 $m = 10;$   
 $Lx = \text{Table}[N[a + i (b - a) / m], \{i, 0, m\}];$ 
```

```
In[57]:= v = Strelba2[f, g, a, b,  $\alpha 1$ ,  $\alpha 2$ ,  $\beta 1$ ,  $\beta 2$ ,  $\gamma 1$ ,  $\gamma 2$ ,  $\epsilon$ ,  $z0$ , Lx];
```

i	z	s
0	1.	
1	0.728782	0.271218
2	0.544211	0.184571
3	0.518902	0.0253084
4	0.518621	0.000281257
5	0.518621	3.26142×10^{-8}

Graf řešení $y_1(x)$

In[58]:= **v[[1]]**Graf řešení $y_2(x)$ In[59]:= **v[[2]]**Tabulka řešení $y_1(x)$ In[60]:= **MatrixForm[v[[3]]]**

Out[60]//MatrixForm=

$$\begin{pmatrix} 0. & 0. \\ 0.1 & 0.0522087 \\ 0.2 & 0.106519 \\ 0.3 & 0.165141 \\ 0.4 & 0.230522 \\ 0.5 & 0.305505 \\ 0.6 & 0.393563 \\ 0.7 & 0.499173 \\ 0.8 & 0.628465 \\ 0.9 & 0.790494 \\ 1. & 1. \end{pmatrix}$$
Tabulka řešení $y_2(x)$

In[61]:= **MatrixForm**[v[[4]]]

Out[61]//MatrixForm=

$$\begin{pmatrix} 0. & 0.518621 \\ 0.1 & 0.529038 \\ 0.2 & 0.560825 \\ 0.3 & 0.61567 \\ 0.4 & 0.69665 \\ 0.5 & 0.80874 \\ 0.6 & 0.959788 \\ 0.7 & 1.16243 \\ 0.8 & 1.43809 \\ 0.9 & 1.82606 \\ 1. & 2.40694 \end{pmatrix}$$

$\alpha = 5.0$

Definice parametrů diferenciální rovnice

In[62]:= **$\alpha = 5.0$;**

Definice pravé strany diferenciální rovnice

In[63]:= **f[x_, y1_, y2_] = y2;**
g[x_, y1_, y2_] = α Sinh[α y1];

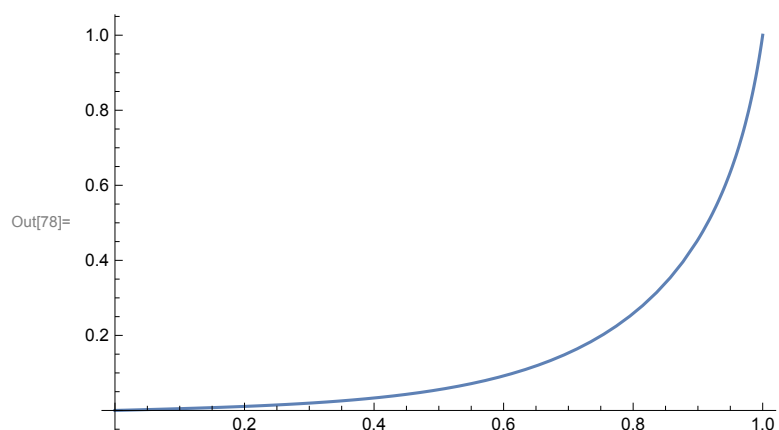
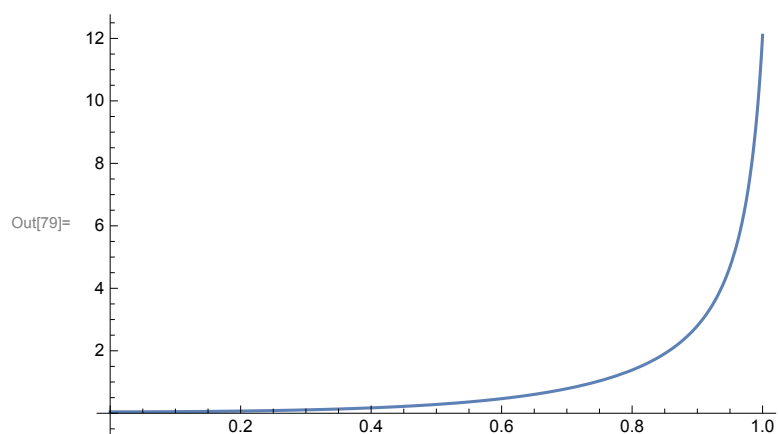
Parametry programu Strelba2

In[65]:= **a = 0.0;**
b = 1.0;
 $\alpha 1 = 1$;
 $\alpha 2 = 1$;
 $\beta 1 = 0$;
 $\beta 2 = 0$;
 $\gamma 1 = 0$;
 $\gamma 2 = 1$;
 $\epsilon = 0.000001$;
z0 = 0.05;
m = 10;
Lx = Table[N[a + i (b - a) / m], {i, 0, m}];

In[77]:= **v = Strelba2[f, g, a, b, $\alpha 1$, $\alpha 2$, $\beta 1$, $\beta 2$, $\gamma 1$, $\gamma 2$, ϵ , z0, Lx];**

i	z	s
0	0.05	
1	0.0470633	0.00293671
2	0.0458541	0.00120919
3	0.0457511	0.000103044
4	0.0457505	6.04957×10^{-7}

Graf řešení $y_1(x)$

In[78]:= **v[[1]]**Graf řešení $y_2(x)$ In[79]:= **v[[2]]**Tabulka řešení $y_1(x)$ In[80]:= **MatrixForm[v[[3]]]**

Out[80]//MatrixForm=

0.	0.
0.1	0.00476808
0.2	0.0107534
0.3	0.0194853
0.4	0.0332005
0.5	0.0554374
0.6	0.0920445
0.7	0.153162
0.8	0.258217
0.9	0.455061
1.	1.

Tabulka řešení $y_2(x)$

In[81]:= **MatrixForm**[**v**[[4]]]

Out[81]//MatrixForm=

$$\begin{pmatrix} 0. & 0.0457505 \\ 0.1 & 0.0515897 \\ 0.2 & 0.0706024 \\ 0.3 & 0.107669 \\ 0.4 & 0.172376 \\ 0.5 & 0.281814 \\ 0.6 & 0.466544 \\ 0.7 & 0.785992 \\ 0.8 & 1.3834 \\ 0.9 & 2.79924 \\ 1. & 12.1006 \end{pmatrix}$$

$\alpha = 10.0$

Definice parametrů diferenciální rovnice

In[82]:= **$\alpha = 10.0$;**

Definice pravé strany diferenciální rovnice

In[83]:= **f[x_, y1_, y2_] = y2;**
g[x_, y1_, y2_] = α Sinh[α y1];

Parametry programu Strelba2

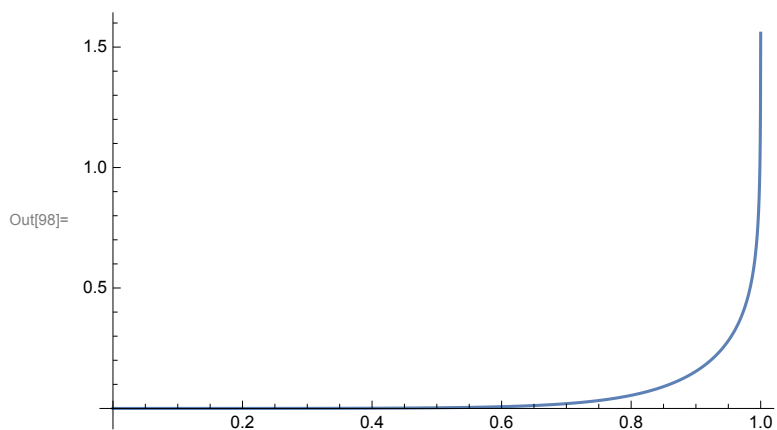
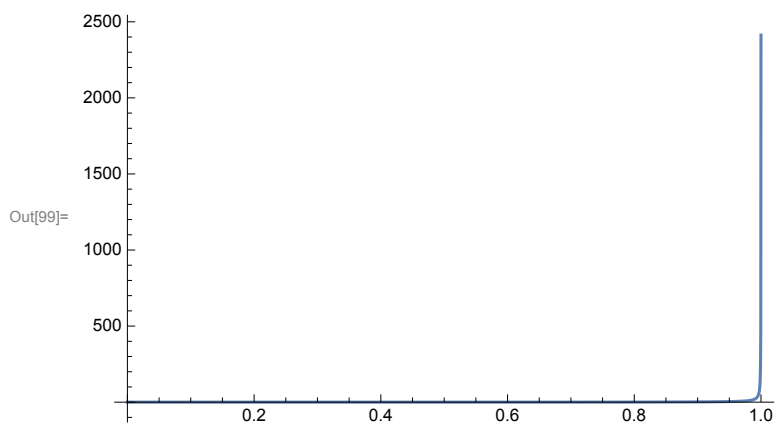
In[85]:= **a = 0.0;**
b = 1.0;
 $\alpha 1 = 1$;
 $\alpha 2 = 1$;
 $\beta 1 = 0$;
 $\beta 2 = 0$;
 $\gamma 1 = 0$;
 $\gamma 2 = 1$;
 $\epsilon = 0.000001$;
z0 = 0.00035;
m = 10;
Lx = Table[N[a + i (b - a) / m], {i, 0, m}];

Museli jsme zvolit hodně přesný nástřel

In[97]:= **v = Strelba2[f, g, a, b, $\alpha 1$, $\alpha 2$, $\beta 1$, $\beta 2$, $\gamma 1$, $\gamma 2$, ϵ , z0, Lx];**

i	z	s
0	0.00035	
1	0.000363094	0.0000130944
2	0.000362691	4.03278×10^{-7}

Graf řešení $y_1(x)$

In[98]:= **v[[1]]**Graf řešení $y_2(x)$ In[99]:= **v[[2]]**Tabulka řešení $y_1(x)$ In[100]:= **MatrixForm[v[[3]]]**

Out[100]/MatrixForm=

0.	0.
0.1	0.0000426505
0.2	0.000131616
0.3	0.000363544
0.4	0.000990347
0.5	0.00269286
0.6	0.00732095
0.7	0.0199147
0.8	0.0544226
0.9	0.154246
1.	1.55774

Tabulka řešení $y_2(x)$

```
In[101]:= MatrixForm[v[[4]]]
```

```
Out[101]/MatrixForm=
```

$$\begin{pmatrix} 0. & 0.000362691 \\ 0.1 & 0.000559877 \\ 0.2 & 0.00136523 \\ 0.3 & 0.00365349 \\ 0.4 & 0.00991015 \\ 0.5 & 0.0269318 \\ 0.6 & 0.0732268 \\ 0.7 & 0.199477 \\ 0.8 & 0.550967 \\ 0.9 & 1.69999 \\ 1. & 2413.22 \end{pmatrix}$$

$\alpha = 20.0$

Definice parametrů diferenciální rovnice

```
In[102]:=  $\alpha = 20.0;$ 
```

Definice pravé strany diferenciální rovnice

```
In[103]:= f[x_, y1_, y2_] = y2;  
g[x_, y1_, y2_] =  $\alpha$  Sinh[ $\alpha$  y1];
```

Parametry programu Strelba2

```
In[105]:= a = 0.0;  
b = 1.0;  
 $\alpha 1 = 1;$   
 $\alpha 2 = 1;$   
 $\beta 1 = 0;$   
 $\beta 2 = 0;$   
 $\gamma 1 = 0;$   
 $\gamma 2 = 1;$   
 $\epsilon = 0.000001;$   
z0 = 0.000000050421;  
m = 10;  
Lx = Table[N[a + i (b - a) / m], {i, 0, m}];
```

```
In[117]:= v = Strelba2[f, g, a, b, α1, α2, β1, β2, γ1, γ2, ε, z0, Lx];
```

```
      i      z      s
```

```
0      5.0421×10-8
```

... **NDSolve**: At NumMath`DEStrelba`Private`x\$1266 == 0.9441149517538657, step size is effectively zero; singularity or stiff system suspected.

... **InterpolatingFunction**: Input value {1.} lies outside the range of data in the interpolating function. Extrapolation will be used.

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... **InterpolatingFunction**: Input value {1.} lies outside the range of data in the interpolating function. Extrapolation will be used.

... **General**: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

```
1      5.0421×10-8      1.88411×10-18
```

... **NDSolve**: At NumMath`DEStrelba`Private`x\$1266 == 0.9406887520790215, step size is effectively zero; singularity or stiff system suspected.

Nepomohl ani hodně přesný nástřel, úloha je stiff - museli bychom zvolit jinou metodu pro řešení počáteční úlohy

(zde byla použita metoda Runge- Kuttova 4. řádu a ta není vhodná pro stiff úlohy)