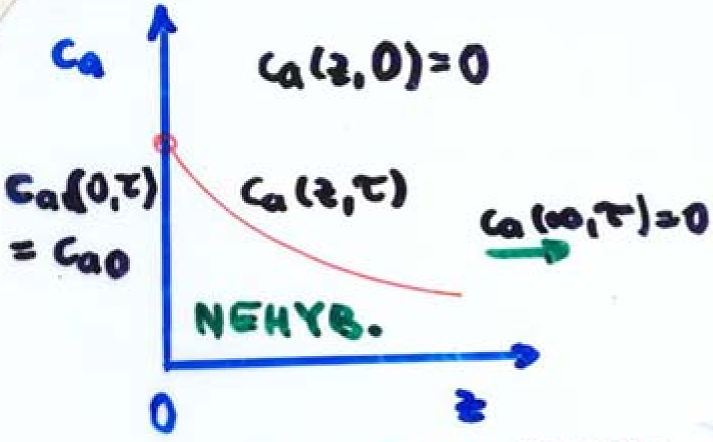



POLONEKON.
VRSTVA

# NEUST. DIFUZE

$$\frac{\partial c_a}{\partial \tau} = D_{ab}\frac{\partial^2 c_a}{\partial z^2}$$

LAPLACE

$$\frac{d^2\hat{c}_a}{dz^2} - \frac{p}{D_a}\hat{c}_a = 0$$

$$m^2 = \frac{p}{D_a}\ , \quad m = \pm\sqrt{\frac{p}{D_a}}$$

$$\hat{c}_a = B_1 \exp\left(z\sqrt{\frac{p}{D_a}}\right) + B_2 \exp\left(-z\sqrt{\frac{p}{D_a}}\right)$$

$$z\to\infty,\ \hat{c}_a = 0 \Rightarrow 0 = B_1 e^{\infty} + B_2 e^{-\infty} \Rightarrow B_1 = 0$$

$$z=0,\ \hat{c}_a = \frac{c_{a0}}{p} \Rightarrow \frac{c_{a0}}{p} = B_2$$

$$\hat{c}_a = \frac{c_{a0}}{p}\exp\left(-z\sqrt{\frac{p}{D_a}}\right)$$

$$\mathcal{L}^{-1}\left[\frac{1}{p}e^{-b\sqrt{p}}\right] = \operatorname{erfc}\frac{b}{2\sqrt{\tau}}$$

$$b = z/\sqrt{D_a}$$

$$\operatorname{erfc} x = 1 - \operatorname{erf} x = 1 - \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du$$

$$\boxed{\frac{c_a}{c_{a0}} = \operatorname{erfc}\frac{z}{2\sqrt{D_a\tau}}}$$

$$\hat{j}_{a0} = -D_a \left.\frac{d\hat{c}_a}{dz}\right|_0 = D_a\frac{c_{a0}}{p}\sqrt{\frac{p}{D_a}} = c_{a0}\sqrt{\frac{D_a}{p}}$$

$$\mathcal{L}^{-1}\left[\frac{1}{\sqrt{p}}\right] = \frac{1}{\sqrt{\pi\tau}}$$

$$\boxed{j_{a0} = c_{a0}\sqrt{\frac{D_a}{\pi\tau}}}$$

$$\frac{1}{\tau}\int_0^\tau \tau^{-1/2}\,d\tau = \frac{2}{\tau}\left[\tau^{\frac{1}{2}}\right]_0^\tau$$

$$\bar{j}_{a0} = \frac{1}{\tau}\int_0^\tau j_{a0}\,d\tau = 2c_{a0}\sqrt{\frac{D_a}{\pi\tau}}$$

# LAPLACE

STAND. TYP

$$F(p) = \int_0^\infty f(t)\, e^{-pt}\, dt = \mathcal{L}[f(t)] = \hat{f}$$

OBRAZ

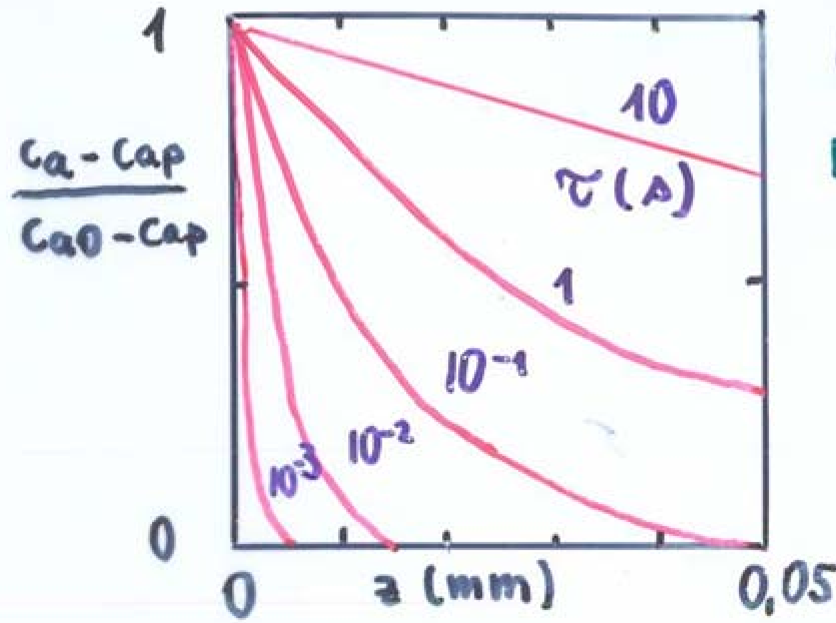
ORIGINÁL (PŘEDMĚT)

1. $\mathcal{L}[f'(t)] = p\hat{f} - f(0+)$

2. $\mathcal{L}\left[\dfrac{\partial f(t,a)}{\partial a}\right] = \dfrac{d\hat{f}(a)}{da} = \hat{f}'$

3. $\mathcal{L}[k f(t)] = k\hat{f}$

4. $\mathcal{L}[k] = \dfrac{k}{p}$

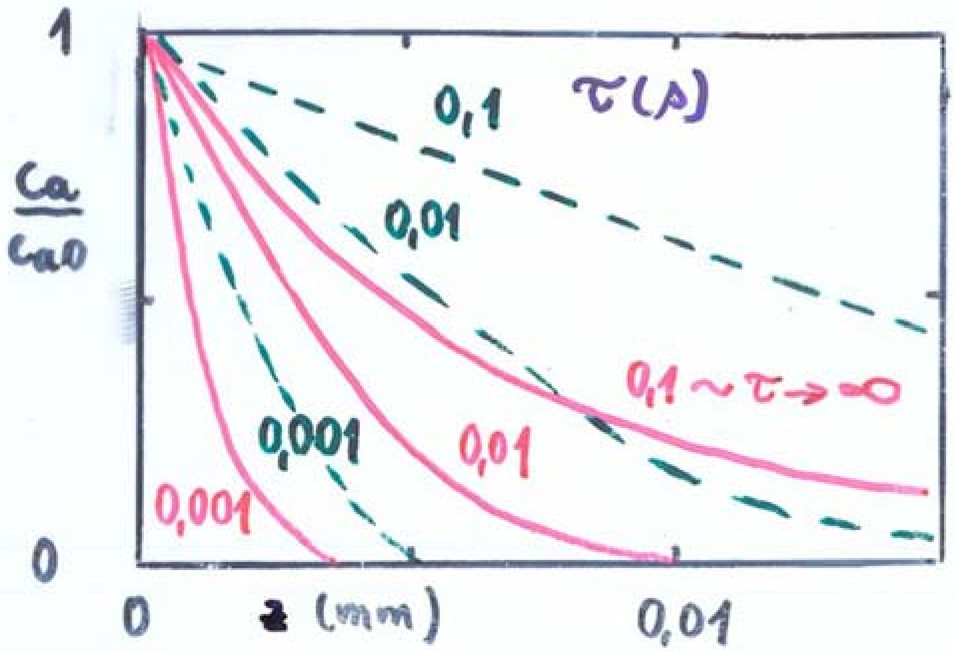



$D_a = 1\cdot 10^{-9}\ m^2 s^{-1}$

$D_a = 1\cdot 10^{-5}\ m^2 s^{-1}$

$z = z \cdot 10^{2}$

$\tau = \tau \cdot 10^{-4}$




$D_a = 2\cdot 10^{-9}\ m^2 s^{-1}$

$k = 1\ s^{-1}$

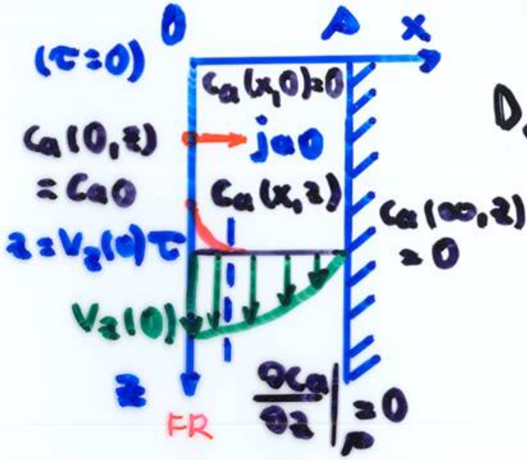
STAC. STAV

$(\tau \to \infty)$

$$\frac{c_a}{c_{a0}} = e^{-z\sqrt{\frac{k}{D_a}}}$$

(erfc $(-\infty) = 2$,
erfc $(+\infty) = 0$)

# ABSORPCE DO STÉKAJÍCÍHO FILMU




PARAB. PROFIL (LAM.)

$$\frac{\partial c_a}{\partial \tau} = 0$$

① ZANEDB. KONVEKT. TRANSP. →

$$D_{ab}\left(\frac{\partial^2 c_a}{\partial x^2} \overset{\gg}{+} \cancel{\frac{\partial^2 c_a}{\partial z^2}}\right) = \cancel{v_x \frac{\partial c_a}{\partial x}} \overset{\ll}{+} v_z \frac{\partial c_a}{\partial z} + c_a\left(\cancel{\frac{\partial v_x}{\partial x}} + \cancel{\frac{\partial v_z}{\partial z}}\right)$$

| | | |
|---|---|---|
| $x = 0$ | $z > 0$ | $c_a = c_{a0}$ |
| ~~$x = \Delta$~~ | ~~$z \geq 0$~~ | ~~$\frac{\partial c_a}{\partial x} = 0$~~ |
| $x > 0$ | $z = 0$ | $c_a = 0$ |

$$v_z(x) = v_z(0)\left[1 - \cancel{\left(\frac{x}{\Delta}\right)^2}\right]$$

② USTÁL. RYCHL. POLE NESTLAČ. KAP.

③ RYCHLOST MOL.T. ↓ << KONV. ↓

④ HLOUBKA PENETRACE MALÁ

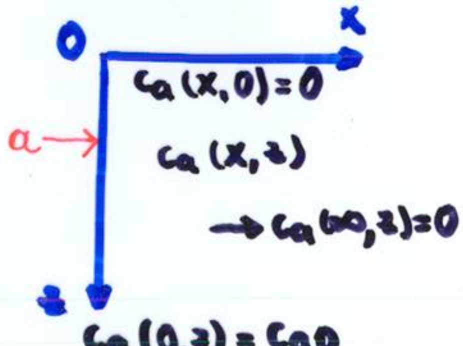
$x \to \infty \quad z \geq 0 \quad c_a = 0$

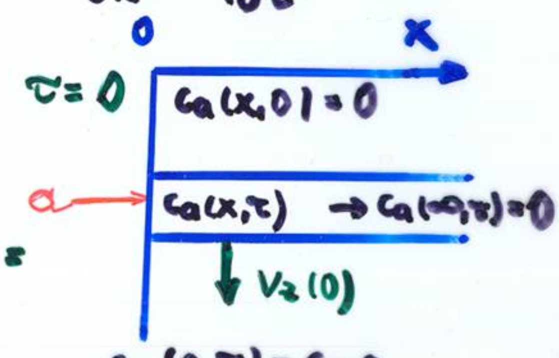
$$D_{ab}\frac{\partial^2 c_a}{\partial x^2} = v_z(0)\frac{\partial c_a}{\partial z} \quad \longleftrightarrow \quad D_{ab}\frac{\partial^2 c_a}{\partial x^2} = \frac{\partial c_a}{\partial \tau}$$

$$\tau = \frac{z}{v_z(0)}$$

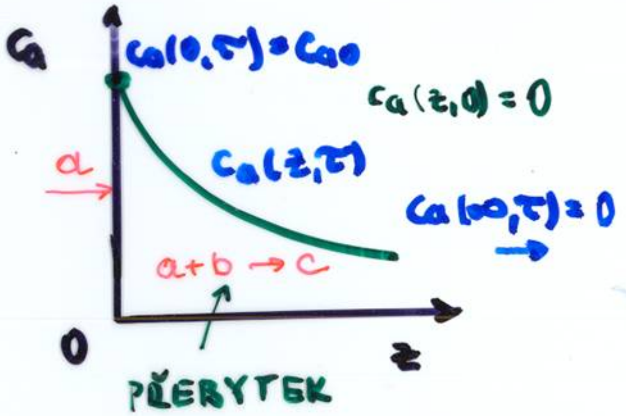
$$\frac{dz}{d\tau} = v_z(0)$$

$$v_z(0)\frac{\partial c_a}{\partial z} = \frac{\partial c_a}{\partial z}\frac{dz}{d\tau} = \frac{\partial c_a}{\partial \tau}$$



# NEUST. DIF. + CH.R.




$$\frac{\partial c_a}{\partial \tau} - D_a \frac{\partial^2 c_a}{\partial z^2} = -\underbrace{k^{+} c_{bs}}_{k} c_a$$

$$p\hat{c} - D_a \hat{c}'' = -k\hat{c}$$

$$m^2 = \frac{p+k}{D_a} \quad , \quad m_{1,2} = \pm\sqrt{\frac{p+k}{D_a}}$$

$$\hat{c} = B_1 e^{m_1 z} + B_2 e^{m_2 z}$$

$$z = 0 \;, \quad \hat{c} = \frac{c_{a0}}{p}$$

$$z \to \infty \;, \quad \hat{c} = 0$$

$$\frac{c_{a0}}{p} = B_1 + B_2 \;\Rightarrow\; B_2 = \frac{c_{a0}}{p}$$

$$0 = B_1 e^{\infty} + 0 \Rightarrow B_1 = 0$$

$$\hat{c} = \frac{c_{a0}}{p} \exp\left[-\sqrt{\frac{p+k}{D_a}}\, z\right]$$

$$\hat{j}_{a0} = -D_a \left.\frac{d\hat{c}}{dz}\right|_0 = \frac{c_{a0}\sqrt{D_a(p+k)}}{p}$$

$$\frac{c_a}{c_{a0}} = f\left(z\sqrt{\frac{k}{D_a}} \,,\, \frac{z}{2\sqrt{D_a\tau}} \,,\, \sqrt{k\tau}\right)$$

$$j_{a0} = f\left(\sqrt{k\tau}\right)$$

$$\frac{c_a}{c_{a0}} = 0{,}5\left[\exp\left(-z\sqrt{\frac{k}{D_a}}\right)\mathrm{erfc}\left(\frac{z}{2\sqrt{D_a\tau}} - \sqrt{k\tau}\right) + \exp\left(z\sqrt{\frac{k}{D_a}}\right)\mathrm{erfc}\left(\frac{z}{2\sqrt{D_a\tau}} + \sqrt{k\tau}\right)\right]$$

$$j_{a0} = c_{a0}\sqrt{D_a k}\left[\mathrm{erf}\sqrt{k\tau} + \frac{\exp(-k\tau)}{\sqrt{\pi k\tau}}\right]$$

$$\bar{j}_{a0} = \frac{1}{\tau}\int_0^{\tau} j_{a0}\, d\tau = c_{a0}\sqrt{D_a k}\left[\left(1 + \frac{1}{2k\tau}\right)\mathrm{erf}\sqrt{k\tau} + \frac{\exp(-k\tau)}{\sqrt{\pi k\tau}}\right]$$

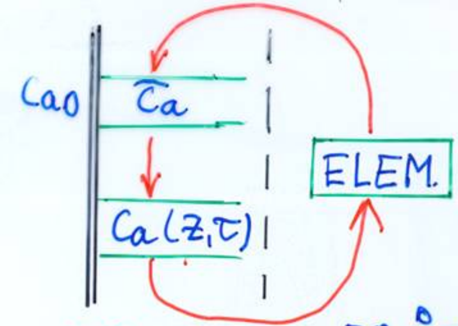
$\sim 1$ $\qquad$ $\sim 0$ $\qquad$ $k\tau > 2{,}8$

$$\bar{k}_L = 2\sqrt{\frac{D_a}{\pi\tau}} \quad , \quad k\tau = \frac{4}{\pi}\frac{D_a k^{+} c_{bs}}{k_L^2} = \frac{4}{\pi} H^2$$

# PŘESTUP

$j_{a0} = k_c (c_{a0} - \bar{c}_a)$

## PENETR. T. (HIGBIE)




FR JAKO PRŮT. SYSTÉM

PÍST. TOK

KONST. DOBA PRODL. $\tau = \dfrac{4 D_a}{\pi \bar{k}_c^2}$

$k_c = \sqrt{\dfrac{D_a}{\pi \tau}}$ , $\bar{k}_c = 2\sqrt{\dfrac{D_a}{\pi \tau}}$

## T. OBNOV. POVRCHU (DANCKWERTS)

* MÍSIČ , FREKV. FCE $\phi = \dfrac{1}{\bar{\tau}} \exp\left(-\dfrac{\tau}{\bar{\tau}}\right)$

KONST. FREKV. VÝM. $p = \dfrac{1}{\bar{\tau}} = \dfrac{\bar{k}_c^2}{D_a}$

$$\bar{k}_c = \int_0^\infty k_c(t)\phi(t)\,dt = p\int_0^\infty k_c(t)\, e^{-pt}\,dt = p\,\hat{k}_c = p\sqrt{\frac{D_a}{p}} = \sqrt{p D_a}$$

HIG.

## CH.R.

$$E = \frac{j_{a0}}{\bar{k}_c c_{a0}} = \frac{\sqrt{D_a k}}{\bar{k}_c}\left[\left(1 + \frac{1}{k\tau}\right)\operatorname{erf}\sqrt{k\tau} + \frac{\exp(-k\tau)}{\pi k \tau}\right], \quad k = k^+ c_{b0}$$

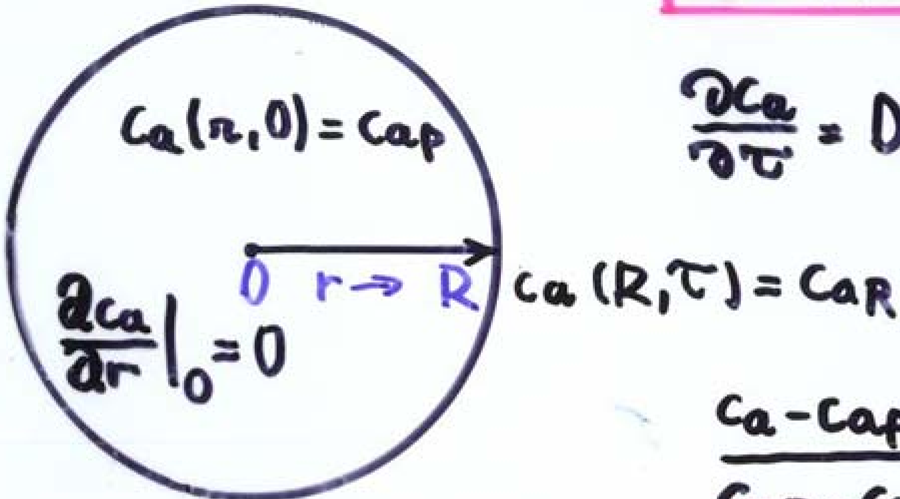
HIG. $H^2 = \dfrac{D_a k}{\bar{k}_c^2} = \dfrac{\pi k \tau}{4} \Rightarrow E = H\,[f(H)]$

FILM $(\tau \to \infty)$, $\operatorname{erf}(\infty) = 1 \Rightarrow E = \sqrt{\dfrac{D_a k}{\bar{k}_c^2}} = H$

DAN. $\bar{j}_{a0} = p\displaystyle\int_0^\infty j_{a0}\, e^{-pt}\,dt = p\,\hat{j}_{a0} = c_{a0}\sqrt{D_a (p+k)}$

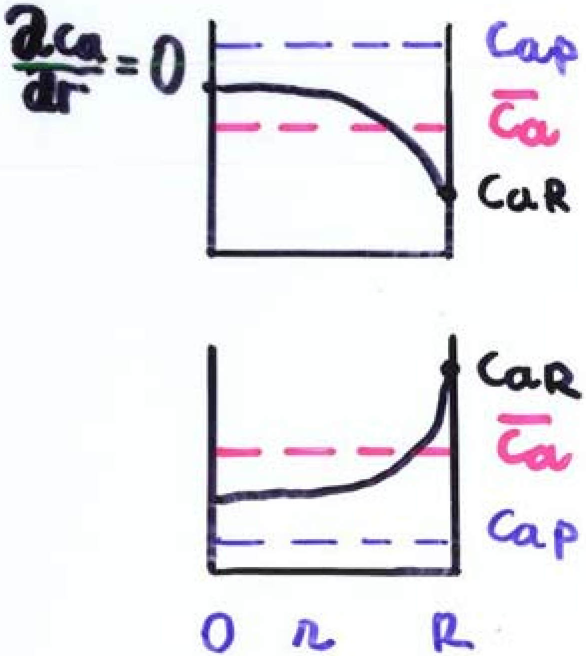
$$E = \sqrt{\frac{D_a (p+k)}{D_a p}} = \sqrt{1 + \frac{k}{p}} = \sqrt{1 + \frac{D_a k}{\bar{k}_c^2}} = \sqrt{1 + H^2}$$

# KOULE




$$\frac{\partial c_a}{\partial \tau} = D_a \left( \frac{\partial^2 c_a}{\partial r^2} + \frac{2}{r} \frac{\partial c_a}{\partial r} \right)$$

$$\frac{c_a - c_{ap}}{c_{aR} - c_{ap}} = f\left( \frac{r}{R}, \frac{D_a \tau}{R^2} \right)$$

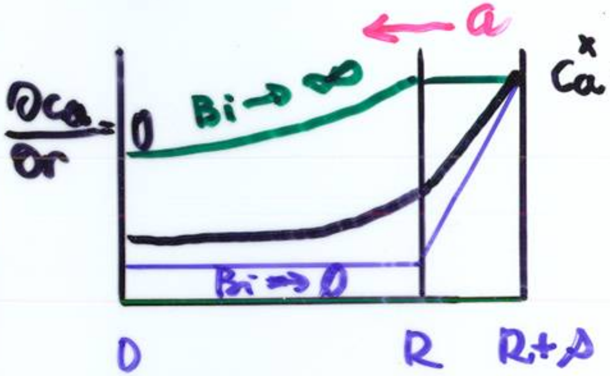



$$\frac{\overline{c_a} - c_{ap}}{c_{aR} - c_{ap}} = f(Fo)$$

$$= 1 - 6 \sum_{k=1}^{\infty} \frac{1}{k\pi} \exp\left(-k^2 \pi^2 Fo\right)$$

# VNĚJŠÍ ODPOR

$$\frac{\overline{c_a} - c_{aP}}{c_a^* - c_{aP}} = f(Bi, Fo)$$

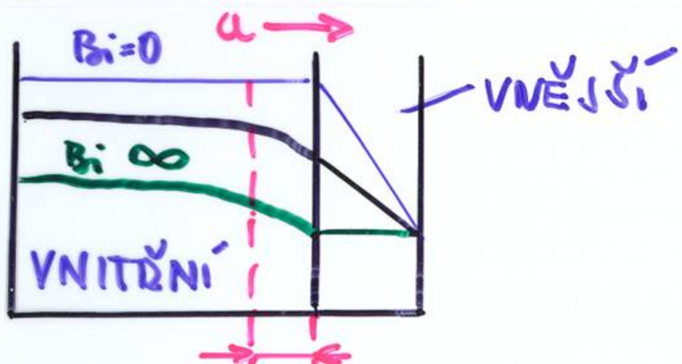

$$c_a^* = K_a \overline{c}'_a$$

$$D_a \frac{\partial c_a}{\partial r}\Big|_R = k'(\overline{c}'_a - c'_{aR})$$

$\overline{c}'_a = \frac{c_a^*}{K_a}$, $c'_{aR} = \frac{c_{aR}}{K_a}$

$$\frac{\partial c_a}{\partial z}\Big|_1 = \frac{k'_c R}{K_a D_a}(c_a^* - c_{aR})$$

$z = r/R$, $\frac{k'_c R}{K_a D_a} = Bi$




ZDÁNLIVÝ FILM (0,1 d)


$\delta \sim 10^{-4}$ m (g)

$\sim 10^{-5}$ m (l)

**Bi → ∞** ($k'_c \to \infty$)

odpor vnitř. dif.

(Bi > 30)

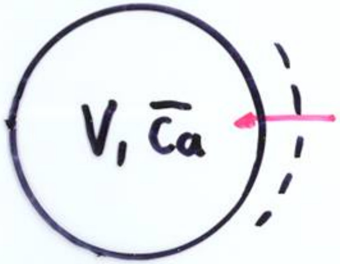
**Bi = 0**, $c_a = \overline{c_a}$

$K_a D_a \gg k'_c$ (PROMÍCH.)

odpor vnější dif.

(Bi < 0,1)

## $Bi = 0$




$$j_a = k'_c(\bar{c}'_a - c'_{aR}) = \frac{k'_c}{K_a}(c_a^{*} - \bar{c}_a)$$

konst.

$$V\frac{d\bar{c}_a}{d\tau} = j_a A$$

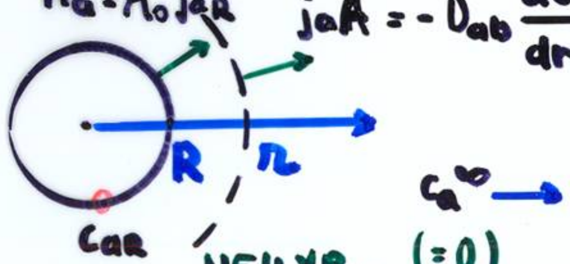
$\frac{4}{3}\pi R^3$ — $4\pi R^2$

$$-\ln\frac{c_a^{*} - \bar{c}_a}{c_a^{*} - c_{ap}} = \frac{3k'_c\tau}{RK_a}\cdot\frac{R}{R}\cdot\frac{D_a}{D_a} = 3\,Bi\,Fo$$

## USTÁL. DIF. VNĚ

$\dot{n}_a = A_0 j_{aR}$

$$j_a A = -D_{ab}\frac{dc_a}{dr}A$$




$$\frac{k_c R}{D_{ab}} = 1 \qquad Sh = \frac{k_c d}{D_{ab}} = 2$$

$$Sh = 2 + a\,Re^{1/2}\,Sc^{1/3}$$

$\sim 0{,}6$

$$\dot{n}_a = j_{aR}A_0 = -D_{ab}A\frac{dc_a}{dr} \quad (= \text{konst})$$

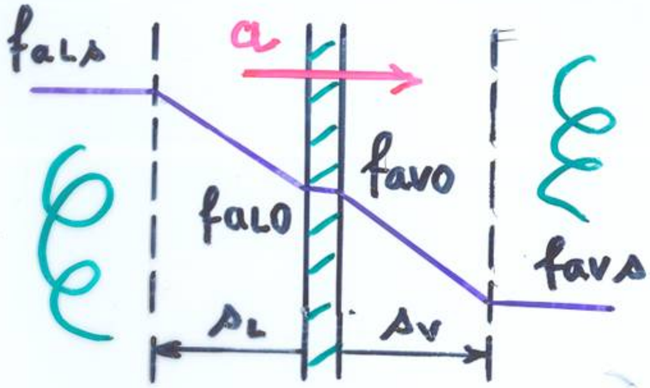
$4\pi R^2$ — $4\pi r^2$

$$j_{aR}R^2 = -D_{ab}r^2\frac{dc_a}{dr}$$

$$\int_R^{\infty}\frac{dr}{r^2} = -\frac{D_{ab}}{j_{aR}R^2}\int_{c_{aR}}^{c_a^{\infty}}dc_a$$

$$j_{aR}R = D_{ab}(c_{aR} - c_a^{\infty})$$

$k_c(c_{aR} - c_a^{\infty})$




$$j_{ao} = F_L\,(f_{aLs} - f_{aLo}) = F_o\,(f_{aLo} - f_{avo}) = F_v\,(f_{avo} - f_{av})$$

$$F_o \to \infty \,,\; f_{aLo} = f_{avo}$$

$$\left.\begin{aligned} \frac{j_{ao}}{F_L} &= f_{aLs} - f_{aLo} \\ \frac{j_{ao}}{F_v} &= f_{avo} - f_{avs} \end{aligned}\right\} \quad j_{ao}\underbrace{\left(\frac{1}{F_L} + \frac{1}{F_v}\right)}_{\frac{1}{F}} = f_{aLs} - f_{avs}$$

$$f_a = \gamma_a\, x_a\, f_a^{o}$$

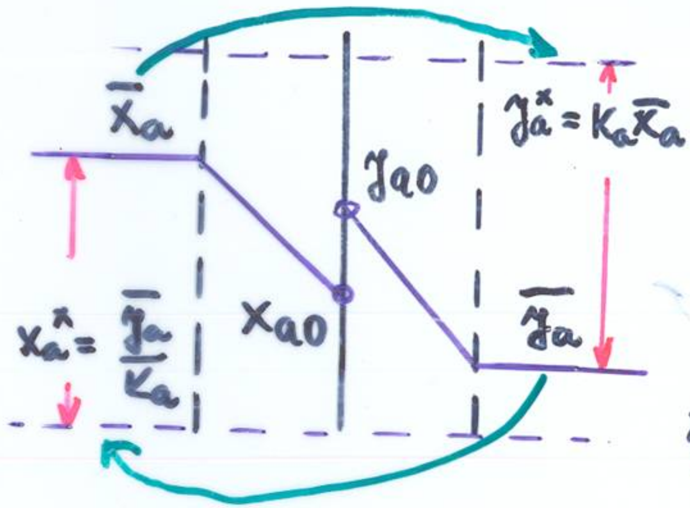
$$j_{ao} = \underbrace{F_L\, \gamma_{aL}\, f_{aL}^{o}}_{k_x}\,(\bar{x}_a - x_{ao}) = \underbrace{F_v\, \gamma_{av}\, f_{av}^{o}}_{k_y}\,(y_{ao} - \bar{y}_a)$$

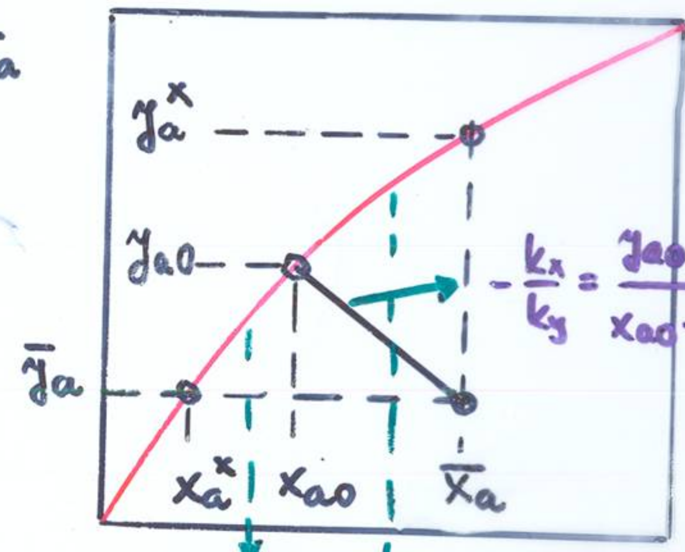
$$x_a = \frac{c_a}{c} \quad , \quad y_a = \frac{p_a}{p}$$

$$j_{ao} = k_c\,(\bar{c}_a - c_{ao}) = k_p\,(p_{ao} - \bar{p}_a)$$

$$j_{ao} = k_x(\bar{x}_a - x_{ao}) = k_y(y_{ao} - \bar{y}_a)$$

## PŘESTUP




$$-\frac{k_x}{k_y} = \frac{y_{ao} - \bar{y}_a}{x_{ao} - \bar{x}_a}$$

## PROSTUP

$$m_x = \frac{y_{ao} - \bar{y}_a}{x_{ao} - x_a^*} \qquad m_y = \frac{y_a^* - y_{ao}}{\bar{x}_a - x_{ao}}$$

$$\frac{j_{ao}}{k_x} = \bar{x}_a - x_{ao} \qquad \frac{j_{ao}}{k_y} = y_{ao} - \bar{y}_a$$

$$\frac{j_{ao}}{m_x k_y} = x_{ao} - x_a^* \qquad \frac{j_{ao} m_y}{k_x} = y_a^* - y_{ao}$$

$$j_{ao} = K_x(\bar{x}_a - x_a^*) \qquad j_{ao} = K_y(y_a^* - \bar{y}_a)$$

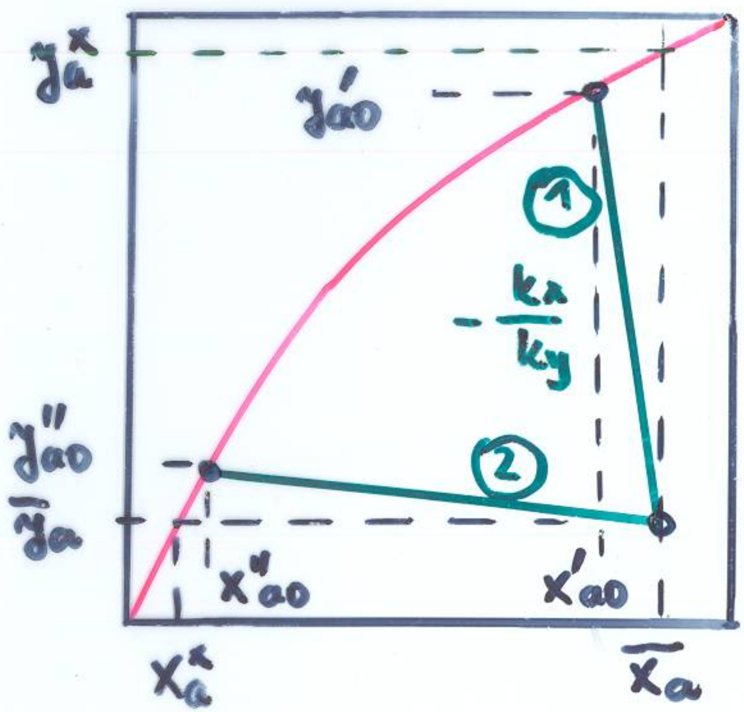
$$\frac{1}{K_x} = \frac{1}{k_x} + \frac{1}{m_x k_y} \qquad \frac{1}{K_y} = \frac{m_y}{k_x} + \frac{1}{k_y}$$

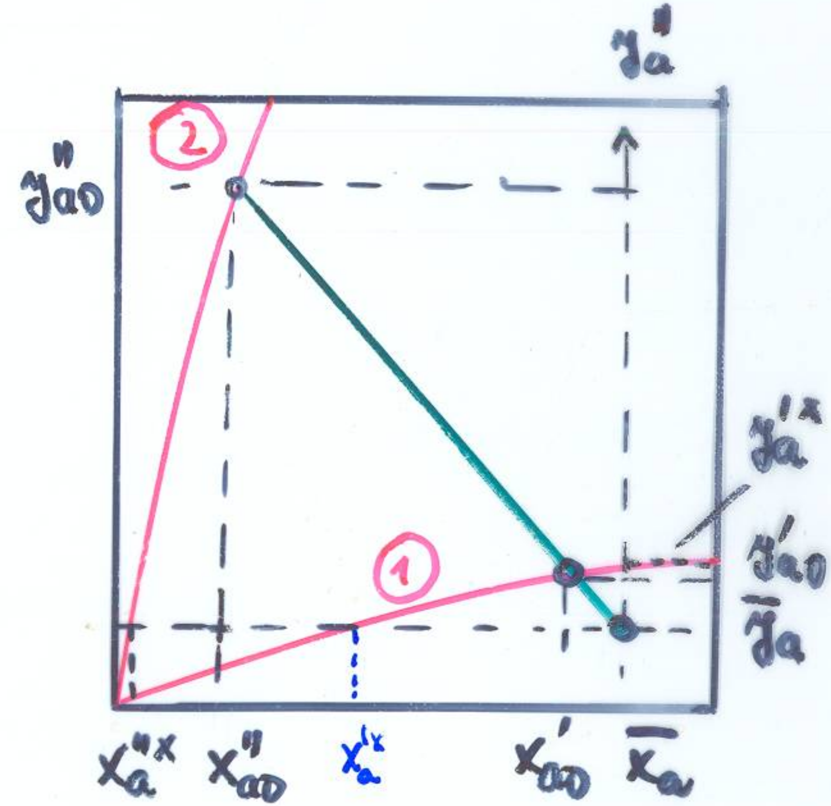
$$\frac{1}{K_x} = \frac{1}{k_x} + \frac{1}{m_x k_y} \qquad\qquad \frac{1}{K_y} = \frac{m_y}{k_x} + \frac{1}{k_y}$$

2 2 $\quad \gg \quad K_x = k_x \quad (y_{a0} = y_a^*)$

1 1 $\quad \ll \quad K_y = k_y \quad (x_a^* = x_{a0})$

$$m_x = m_y = m : \quad K_x = m K_y$$



# TURBULENTNÍ DIFÚZE

$c_i = \bar{c}_i + c'_i$ $\quad \xrightarrow{\text{zprům.}} \quad \overline{c_i} = \overline{\bar{c}_i} + \overline{c'_i}$

$\vec{v}_m = \bar{\vec{v}}_m + \vec{v}'_m$ $\qquad \bar{c}_i = \bar{c}_i + 0$

R.K.: $$\frac{\partial(\bar{c}_a + \overline{c'_a})}{\partial \tau} + \nabla\left[(\bar{c}_a + c'_a)(\bar{\vec{v}}_m + \vec{v}'_m)\right] - D_{ab}\nabla^2(\bar{c}_a + \overline{c'_a}) = -k\,(\bar{c}_a + \overline{c'})$$

$$-k\,\overline{(\bar{c}_a + c')^2}$$

$$\nabla\left[\overline{\bar{c}_a \bar{\vec{v}}_m} + \underbrace{\overline{\bar{c}_a \vec{v}'_m} + \overline{c'_a \bar{v}_m}}_{0} + \underbrace{\overline{c'_a \vec{v}'_m}}_{\vec{J}_{Ta}}\right] \qquad -k(\overline{\bar{c}_a^2} + \underbrace{2\overline{\bar{c}_a c'_a}}_{0} + \overline{c'^2})$$

$$\frac{\partial \bar{c}_a}{\partial \tau} + \nabla(\bar{c}_a \bar{\vec{v}}_m + \overline{c'\vec{v}'_m}) - D_{ab}\nabla^2 \bar{c}_a = -k\,\bar{c}_a$$

$$-k(\bar{c}_a^2 + \overline{c'^2})$$

$$+ \nabla\cdot\vec{j}^m_a + \nabla\cdot\vec{J}_{Ta} + \nabla\cdot\vec{J}_{Da} = \sigma_a + \sigma_{Ta}$$

$$\vec{J}_{Ta} = -D_{Ta}\nabla\bar{c}_a$$

$$\vec{j}^m_a = -(D_T + D_{ab})\nabla\bar{c}_a$$

$$\frac{D_T}{D_{ab}} \qquad 10^2\ (g), \quad 10^5\ (l)$$