TURBULENTNÍ DIFÚZE

$c_i = \bar{c}_i + c'_i$ $\xrightarrow{\text{zprům.}}$ $\overline{c_i} = \overline{\bar{c}_i} + \overline{c'_i}$

$\overline{c}_i = \bar{c}_i + 0$

$\vec{v}_m = \bar{\vec{v}}_m + \vec{v}'_m$

R.K.: $\dfrac{\partial(\bar{c}_a + \overline{c'_a})}{\partial \tau} + \nabla\left[\overline{(\bar{c}_a + c'_a)(\bar{\vec{v}}_m + \vec{v}'_m)}\right] - D_{ab}\nabla^2(\bar{c}_a + \overline{c'_a}) = -k(\bar{c}_a + \overline{c'})$
$-k\,\overline{(\bar{c}_a + c')^2}$

$\nabla\left[\overline{\bar{c}_a \bar{\vec{v}}_m} + \underbrace{\overline{\bar{c}_a \vec{v}'_m} + \overline{c'_a \bar{\vec{v}}_m}}_{0} + \underbrace{\overline{c'_a \vec{v}'_m}}_{\vec{J}_{Ta}}\right]$ $\quad -k(\overline{\bar{c}_a^2} + \underbrace{2\,\overline{\bar{c}_a c'_a}}_{0} + \overline{c'^2})$

$\dfrac{\partial \bar{c}_a}{\partial \tau} + \nabla\left(\bar{c}_a \bar{\vec{v}}_m + \overline{c'\vec{v}'_m}\right) - D_{ab}\nabla^2 \bar{c}_a = -k\,\bar{c}_a$
$-k(\bar{c}_a^2 + \overline{c'^2})$

$+\ \nabla\cdot\vec{j}_a^m + \nabla\cdot\vec{J}_{Ta} + \nabla\cdot\vec{J}_{Da} = \sigma_a + \sigma_{Ta}$

$\vec{J}_{Ta} = -D_{Ta}\nabla\bar{c}_a$

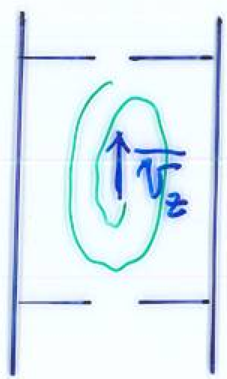
$\vec{j}_a^m = -(D_T + D_{ab})\nabla\bar{c}_a$

$\dfrac{D_T}{D_{ab}}$ $\quad 10^2$ (g), 10^5 (l)

DISPERSNÍ TOK :

SUPERPOSICE : PÍST. TOK + NEUSPOŘ. TOK
(V MAKROMĚŘÍTKU)

VYROVNÁVÁNÍ KONC. PROMÍCH. VE VELKÝCH MĚŘ., SROVNATELNÝCH S ROZMĚRY ZAŘÍZ.



$$\frac{\partial c_i}{\partial \tau} + \bar{v}_z \frac{\partial c_i}{\partial z} - D_z \frac{\partial^2 c_i}{\partial z^2} = \sigma_i$$

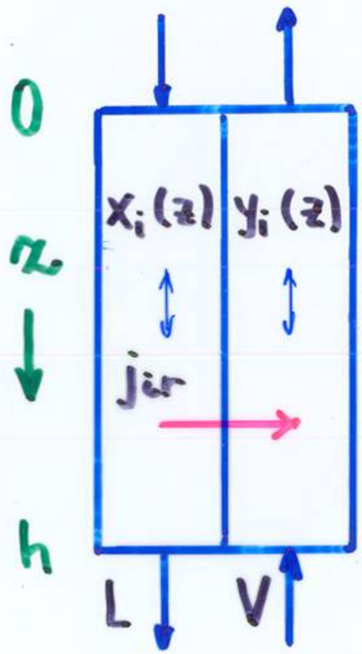
$$\bar{v}_z = \frac{\dot{V}}{S\varepsilon}$$

KOEF. AX. DISPERSE (PODÉL.)

RAD. DISP. (PŘÍČ.)

$$\frac{\partial c_i}{\partial \tau} + \bar{v}_z \frac{\partial c_i}{\partial z} - D_z \frac{\partial^2 c_i}{\partial z^2} - \frac{D_r}{r} \frac{\partial}{\partial r}\left(r \frac{\partial c_i}{\partial r} \right) = \sigma_i$$

USTÁL. ⇌ + AX. DISPERZE



$$j_L = \frac{\dot{n}_L}{\varepsilon \varphi_L S}$$

$$\boxed{\frac{d(j_L x_i)}{dz} - c_L D_L \frac{d^2 x_i}{dz^2} = \sigma_{iL}}$$

$(L \rightarrow V)$

$$\sigma_i = -j_{ir}\, a = \sigma_{iL}\,\varepsilon \varphi_L = -\sigma_{iV}\,\varepsilon \varphi_V$$

$$\boxed{\frac{d(j_V y_i)}{dz} - c_V D_V \frac{d^2 y_i}{dz^2} = \sigma_{iV}}$$

$$-j_V = \frac{\dot{n}_V}{\varepsilon \varphi_V S}$$

$(\varphi_L + \varphi_V = 1)$

$$j_{ir}\, a = k_x a (x_i - x_i^r) = k_y a (y_i^r - y_i) = K_x a (x_i - x_i^*) = K_y a (y_i^* - y_i)$$

$$\boxed{\frac{dx_i}{dZ} - \frac{\varepsilon \varphi_L S c_L D_L}{\dot{n}_L h} \frac{d^2 x_i}{dZ^2} = -\frac{K_x a S h}{\dot{n}_L} (x_i - x_i^*)}$$

$\Longrightarrow$ $Z = \frac{z}{h}$ $\quad \frac{1}{Pe_L} = \frac{D_L}{v_L h}$ $\quad \bar{N}_x = \frac{h}{H_x}$

$$\boxed{-\frac{dy_i}{dZ} - \frac{1}{Pe_V} \frac{d^2 y_i}{dZ^2} = \bar{N}_y (y_i^* - y_i)}$$

PÍST. TOK $\quad Pe \rightarrow \infty$, $D = 0$

* MÍS. $\quad Pe = 0$, $D \rightarrow \infty$

D.P. (DANCKWERTS)

$\dot{n}_{L0}, x_{i0-}, D_L = 0$

$z = Z = 0$

$\dot{n}_{L0}, x_{i0+}, D_L > 0$

$$\dot{n}_{L0}\, x_{i0-} = \dot{n}_{L0}\, x_{i0+} - c_L D_L S_L \frac{dx_i}{dz}\Big|_{0+}$$

$$\Downarrow$$

$$\frac{dx_i}{dZ} - Pe_L (x_i - x_{i0-}) = 0$$

$\dot{n}_{L1}, x_{i1-}, D_L > 0$

$z = h$

$Z = 1$

$\dot{n}_{L1}, x_{i1+}, D_L = 0$

$$x_{i1-} = x_{i1+}$$

$$\dot{n}_{L1}\, x_{i1} - c_L D_L S_L \frac{dx_i}{dz}\Big|_{h-} = \dot{n}_{L1}\, x_{i1}$$

$$\Downarrow$$

$$\frac{1}{Pe_L} \frac{dx_i}{dZ} = 0 \Rightarrow \frac{dx_i}{dZ} = 0$$

$\downarrow \left(Z = \frac{z}{h}\right)$

$\dot{n}_{V0}, y_{i0}, D_V = 0$

$z = Z = 0$

$\dot{n}_{V0}, y_{i0}, D_V > 0$

$$\frac{dy_i}{dZ} = 0$$

$\dot{n}_{V1}, y_{i1-}, D_V > 0$

$z = h$

$Z = 1$

$\dot{n}_{V1}, y_{i1+}, D_V = 0$

$$\frac{dy_i}{dZ} + Pe_V (y_i - y_{i1+}) = 0$$

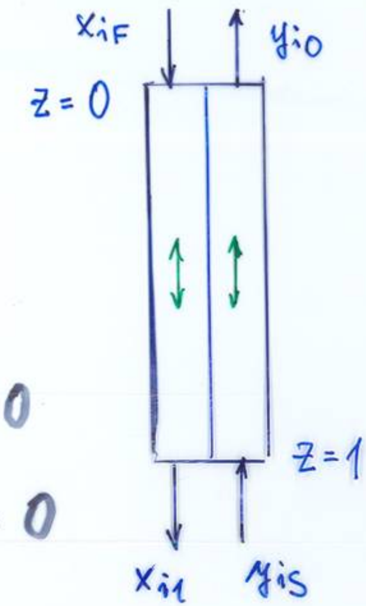
⇌ s ax. disperzí (EXTR.)

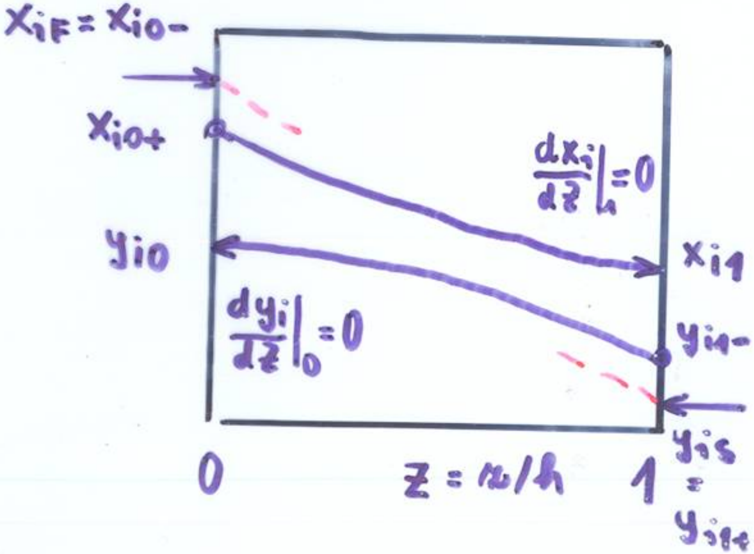
$$\frac{dx_i}{dz} - \frac{1}{Pe_L}\frac{d^2x_i}{dz^2} = -N_x\,(x_i - x_i^*)$$

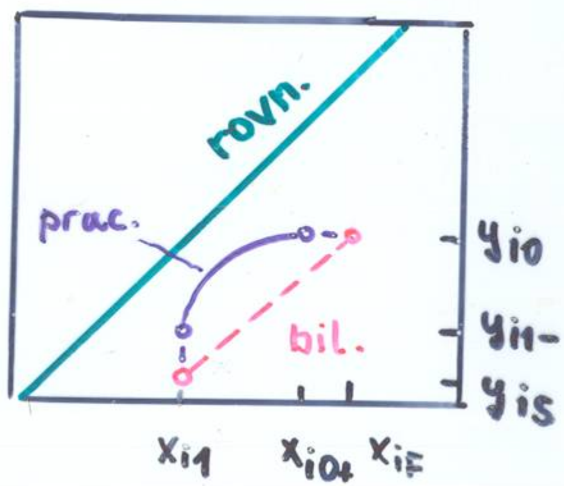
$$-\frac{dy_i}{dz} - \frac{1}{Pe_V}\frac{d^2y_i}{dz^2} = N_y\,(y_i^* - y_i)$$

$z=0$ $\quad \frac{dx_i}{dz} - Pe_L\,(x_i - x_{iF}) = 0\ , \qquad \frac{dy_i}{dz} = 0$

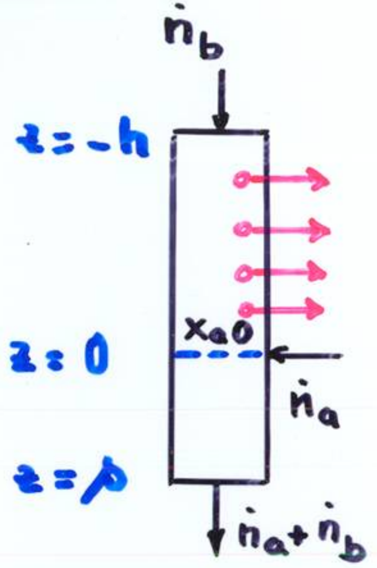
$z=1$ $\quad \frac{dx_i}{dz} = 0\ , \qquad \frac{dy_i}{dz} + Pe_V\,(y_i - y_{iS}) = 0$







STAT. MĚŘ. AX. DISPERSE



$z \le 0$:

$$0 = x_a j - cD \frac{dx_a}{dz}$$

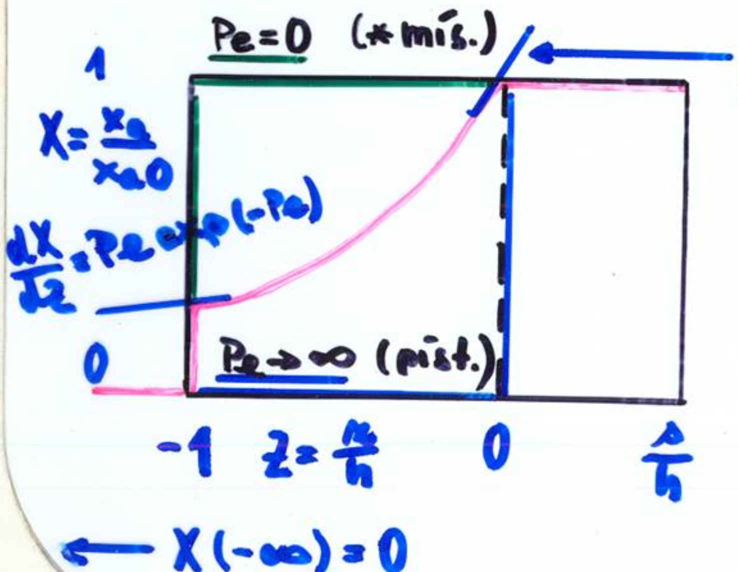
$$j_a = 0 \qquad j = j_a + j_b = j_b \qquad x_{a0} = \frac{j_{a0}}{j_{a0} + j_b} = \frac{\dot{n}_a}{\dot{n}_a + \dot{n}_b}$$

$$\frac{j}{cD} \int_0^z dz = \int_{x_{a0}}^{x_a} \frac{dx_a}{x_a}$$

$$\frac{x_a}{x_{a0}} = \exp\left(\frac{jz}{cD} \cdot \frac{h}{h}\right) = \exp\left(\frac{jh}{cD} \cdot \frac{z}{h}\right)$$

$$X = \exp\left(Pe\, Z\right)$$

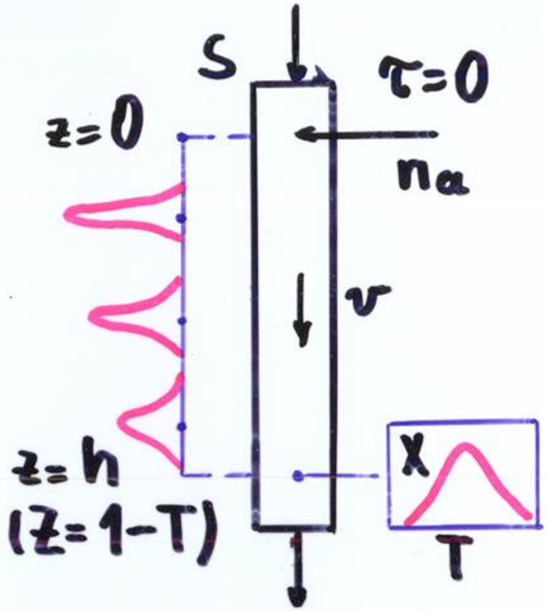
$$\frac{jh}{cD} = \frac{\dot{V}h}{SD} = \frac{vh}{D}$$



$$\frac{dX}{dZ} = Pe \exp(Pe \cdot 0) = Pe$$

$$\frac{dX}{dZ} = Pe \exp\left(Pe\, Z\right)$$

DYNAM. MĚŘ. AX. DISPERSE



$$\frac{\partial c_a}{\partial \tau} + v\frac{\partial c_a}{\partial z} - D\frac{\partial^2 c_a}{\partial z^2} = \frac{n_a}{S}\delta(\tau)\delta(z)$$

$$\delta(y) = 0 \quad y \neq 0\ , \quad \int_{-\infty}^{\infty}\delta(y)\,dy = 1$$

$$Z = \frac{z - v\tau}{h} = \frac{z}{h} - T$$

$$X = \frac{c_a S h}{n_a}\ , \quad Pe = \frac{vh}{D}$$

$$\frac{\partial X}{\partial T} + \frac{1}{Pe}\frac{\partial^2 X}{\partial Z^2} = \delta(T)\delta(Z)$$

$$Z = 0,\ \frac{\partial X}{\partial Z} = 0 \quad \text{(symetrie)}$$

$$Z \to \infty\ , \quad X = 0$$

$$X = \sqrt{\frac{Pe}{4\pi T}}\exp\left[-\frac{Pe\,Z^2}{4T}\right]$$

$$z = h\ ,\ Z = 1 - T\ :$$

$$X = \sqrt{\frac{Pe}{4\pi T}}\exp\left[-\frac{Pe(1-T)^2}{4T}\right]$$

$$\frac{\partial c_a}{\partial \tau} = \frac{\partial c_a}{\partial T}\frac{\partial T}{\partial \tau} + \frac{\partial c_a}{\partial Z}\frac{\partial Z}{\partial \tau} = \frac{\partial c_a}{\partial T}\frac{v}{h} - \frac{\partial c_a}{\partial Z}\frac{v}{h}$$

$$\frac{\partial c_a}{\partial z} = \frac{\partial c_a}{\partial Z}\frac{\partial Z}{\partial z} = \frac{\partial c_a}{\partial Z}\frac{1}{h} \qquad \frac{\partial^2 c_a}{\partial z^2} = \frac{\partial^2 c_a}{\partial Z^2}\frac{1}{h^2}$$

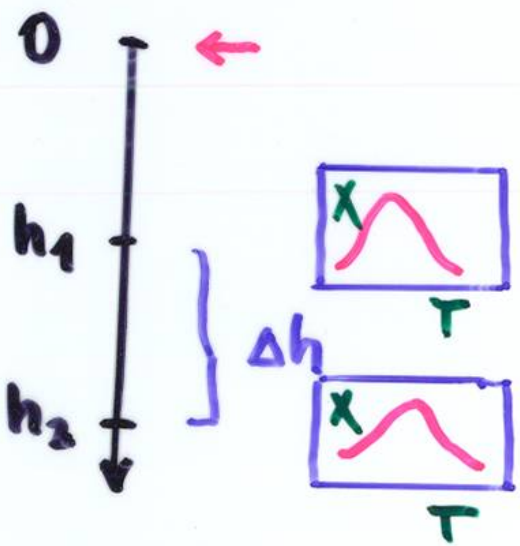
$$\frac{n_a}{S}\delta(\tau)\delta(z) = \frac{n_a}{S}\frac{h}{v}\frac{v}{h^2}\delta(T)\delta(Z)$$

MOMENTY

$$\mu = \frac{\int_0^\infty X(T)\,T\,dT}{\int_0^\infty X(T)\,dT}\,, \quad \sigma^2 = \frac{\int_0^\infty (T-\mu)^2 X(T)\,dT}{\int_0^\infty X(T)\,dT} = \frac{\int_0^\infty X(T)\,T^2\,dT}{\int_0^\infty X(T)\,dT} - \mu^2$$

$$\mu = \frac{2}{Pe} + 1 \qquad \sigma^2 = \frac{8}{Pe^2} + \frac{2}{Pe}$$

$$\mu_\tau = \mu \frac{h}{v} \qquad \sigma_\tau^2 = \sigma^2 \frac{h^2}{v^2}$$



$$T^x = \frac{\tau v}{\Delta h}\,, \quad Pe^x = \frac{v \Delta h}{D}$$

$$\sigma_2^2 - \sigma_1^2 = \frac{2}{Pe^x}$$

PŘEVODOVÉ JEDNOTKY

$$\dot{n}_L \frac{dx_i}{dz} = -k_x a S (x_i - x_i^r)$$

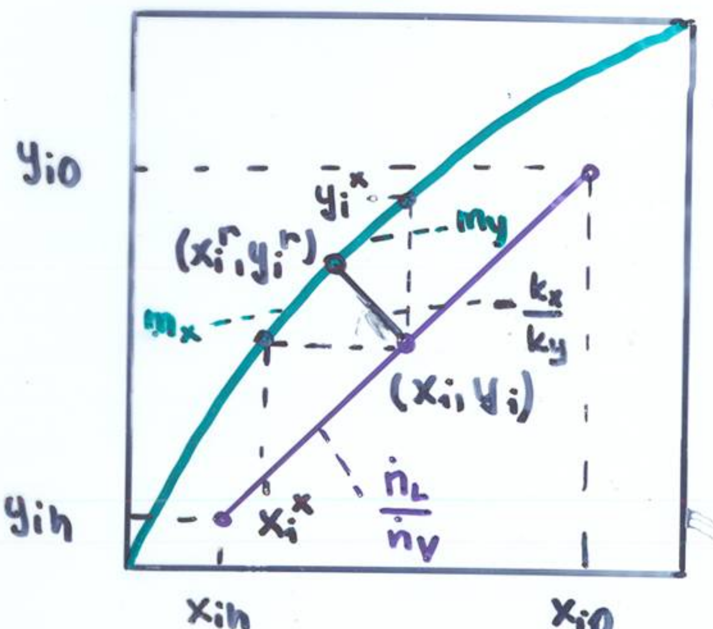
$$n_x = -\int_{x_{i0}}^{x_{ih}} \frac{dx_i}{x_i - x_i^r} = \frac{k_x a S}{\dot{n}_L} \int_0^h dz = \frac{k_x a S}{\dot{n}_L} \ell = \frac{h}{h_x}$$

vnější vnitřní

$$\frac{1}{K_x} = \frac{1}{k_x} + \frac{1}{m_x k_y} \cdot \frac{\dot{n}_V}{\dot{n}_V} \Big/ \frac{\dot{n}_L}{aSh} \qquad \frac{1}{K_y} = \frac{m_y}{k_x} + \frac{1}{k_y}$$

$$H_x = h_x + \frac{\dot{n}_L}{\dot{n}_V m_x} h_y \qquad H_y = \frac{m_y \dot{n}_V}{\dot{n}_L} h_x + h_y$$

$A m_x$ $S m_y$



n_x	n_y	N_x	N_y
x_i	y_i	x_i	y_i
$x_i - x_i^r$	$y_i^r - y_i$	$x_i - x_i^x$	$y_i^x - y_i$
k_x	k_y	K_x	K_y
$\dot{n}_L$	$\dot{n}_V$	$\dot{n}_L$	$\dot{n}_V$
h_x	h_y	H_x	H_y

PJ: PŘECHÁZÍ 1 SLOŽKA

není konst.

$$\frac{d(\dot{n}_L x_a)}{dz} = \dot{n}_L(1-x_a)\frac{d\left(\frac{x_a}{1-x_a}\right)}{dz} = \ldots$$

$\dot{n}_L(1-x_a)\dfrac{x_a}{1-x_a}$

$X_a = \dfrac{x_a}{1-x_a} = x_a^{-a}$ ①

$$\dot{n}_L^{-a}\frac{dX_a}{dz} = -k_X a S(X_a - X_a^r)$$

$$\frac{\dot{n}_L}{1-x_a}\frac{dx_a}{dz} = -k_x a S(x_a - x_a^r) \quad ②$$

$$\frac{d}{dz}\left(\frac{x_a}{1-x_a}\right) = \frac{1}{(1-x_a)^2}\frac{dx_a}{dz}$$

$$k_x = \frac{k_x^0}{(1-x_a)_{ls}} \leftarrow (1-x_a)_{ls} = \frac{x_a - x_a^r}{\ln\frac{1-x_a^r}{1-x_a}} \quad ③$$

$$x_a = \frac{X_a}{1+X_a}, \quad x_a^r = \frac{X_a^r}{1+X_a^r}$$

$\dfrac{\dot{n}_L}{k_x}$ konst. (spíše než k_X)

① $$dz = -\frac{\dot{n}_L^{-a}}{k_X a S}\cdot\frac{dX_a}{X_a - X_a^r}$$

②a ③a

② $$dz = -\frac{\dot{n}_L}{k_x a S}\cdot\frac{dx_a}{(1-x_a)(x_a - x_a^r)}$$

③ $$dz = -\frac{\dot{n}_L}{k_x^0 a S}\cdot\frac{(1-x_a)_{ls}\,dx_a}{(1-x_a)(x_a - x_a^r)} = -\frac{\dot{n}_L}{k_x^0 a S}\cdot\frac{dx_a}{(1-x_a)\ln\frac{1-x_a^r}{1-x_a}}$$

②a $$dz = -\frac{\dot{n}_L}{k_x a S}\cdot\frac{(1+X_a^r)\,dX_a}{X_a - X_a^r}$$

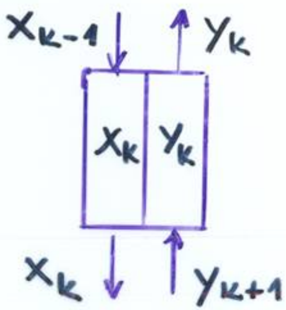
$$\dot{n}_L = n_L^{-a}(1+X_a)$$

③a $$dz = \frac{\dot{n}_L}{k_x^0 a S}\cdot\frac{(1+X_a)_{ls}\,dX_a}{(1+X_a)(X_a - X_a^r)} = -\frac{\dot{n}_L}{k_x^0 a S}\,\frac{dX_a}{(1+X_a)\ln\frac{1+X_a}{1+X_a^r}}$$

$\rightleftarrows$

1. * mích L $\quad Pe_L = 0$
* mích V $\quad Pe_V = 0$

$$N_x = \int_{x_k}^{x_{k-1}} \frac{dx}{x - x^*} \qquad N_y = \int_{y_{k+1}}^{y_k} \frac{dy}{y^* - y}$$



$$\dot{m}_L (x_{k-1} - x_k) = \dot{m}_V (y_k - y_{k+1}) =$$
$$= N_x \dot{m}_L (x_k - x_k^*) = N_y \dot{m}_V (y_k^* - y_k)$$

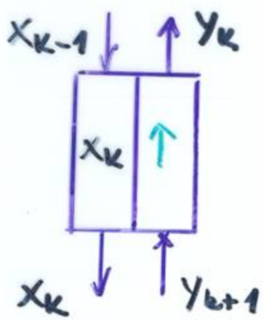
$$N_x = \frac{x_{k-1} - x_k}{x_k - x_k^*} \qquad N_y = \frac{y_k - y_{k+1}}{y_k^* - y_k}$$

$$\frac{1}{E_x^M} = 1 + \frac{1}{N_x} \qquad \frac{1}{E_y^M} = 1 + \frac{1}{N_y}$$

$\frac{1}{E^M} = 1$ pro $N \to \infty$, $K \to \infty$

$$\boxed{E_x^M = \frac{x_{k-1} - x_k}{x_{k-1} - x_k^*}} \qquad \boxed{E_y^M = \frac{y_k - y_{k+1}}{y_k^* - y_{k+1}}}$$

2. * mích L $\quad Pe_L = 0$
píst. V $\quad Pe_V = \infty$



konst

$$-\frac{dy}{dZ} = N_y (y_k^* - y)$$

$$-\int_{y_k}^{y_{k+1}} \frac{dy}{y_k^* - y_k} = N_y \int_0^1 dZ$$

$$\ln \frac{y_k^* - y_{k+1}}{y_k^* - y_k} = N_y \qquad \frac{y_k^* - y_k}{y_k^* - y_{k+1}} = e^{-N_y}$$

$$E_y^M = 1 - e^{-N_y}$$